Root_Polynomial_equidistribution

What can we say about the distribution of roots of a random polynomial?

At first sight very little: given a finite collection of complex numbers we can find a polynomial that has those numbers as roots.

So we may think that the roots of a random polynomial should be randomly distributed.

Let us do an experiment!

We construct polynomials with random coefficients between -5 and 5.

```
from sage.rings.polynomial.complex_roots import complex_roots
x = polygen(ZZ)
@interact
def _raices(n=(3,75,1),u = checkbox(default=False, label='Toggle')):
    p=0
    for i in [0..n]:
        p=x*p+(int(11*random())-5)
    A=Graphics()
    if u:
        A=A+circle((0,0),1,rgbcolor='red')
    a=0
    for P in complex_roots(p,min_prec=.01):
        z=P[0].center()
        if abs(z) > 1:
            a=a+1
        A=A+point(P[0].center())
    A.set_axes_range(-4,4,-2,2)
    A.set_aspect_ratio(1)
    show(A,axes=False,figsize=8)
```

n   3

Toggle ☐
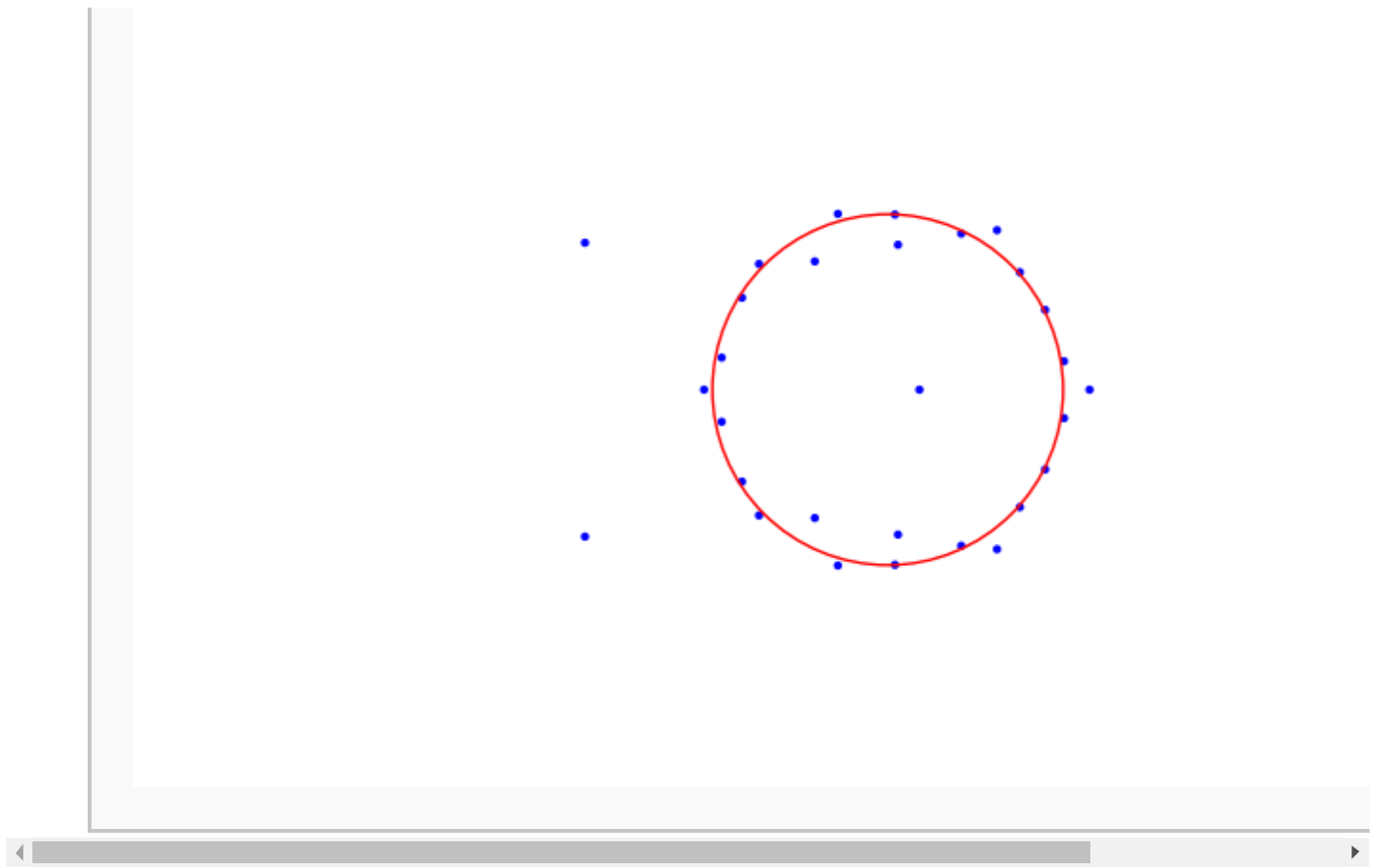
We see that the roots converge to the uniform distribution on the unit circle.