MAXIMAL OPERATORS ASSOCIATED TO CERTAIN GEOMETRIC CONFIGURATIONS

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A problem recently studied in [1] and [2] is the relation between sizes of sets $B, S \subset \mathbb{R}^2$ when B contains the boundary of a square with center in every point of S and sides parallel to the axis. More generally, $B, S \subset \mathbb{R}^n$ when B contains k-skeletons of n-dimensional cubes around every point of S. All this type of problems have associated a maximal operator. For this geometric configuration, we consider the following:

$$\mathcal{M}_\delta^k f(x) = \sup_{1 \le r \le 2} \min_{j=1}^{N} \frac{1}{\mathcal{L}(S_{k,\delta}^j(x,r))} \int_{S_{k,\delta}^j(x,r)} f(y)dy,$$

where $S_{k,\delta}^j(x,r)$ is a δ-neighborhood of each side of a k-skeleton $S_k(x,r)$ with center x and side length 2r. In this work we present results for the behavior of $\mathcal{M}_\delta^k : L^p \to L^p$ when δ tends to 0, for all $1 < p \le \infty$. The bounds found recover, in particular, some results from [2].

[1] T. Keleti, D. Nagy and P. Shmerkin. Squares and their centers. J. Anal. Math., To appear, 2015.

[2] R. Thornton. Cubes and their Centers. Preprint, available at http://arxiv.org/abs/1502.02187, 2017.

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