Embeddings between Grand, Small and Variable Lebesgue Spaces

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We consider the relationship between three Banach function spaces that generalize the classical Lebesgue spaces; Generalized Grand Lebesgue spaces $L^{p),\theta}(\Omega)$, Small Lebesgue spaces $L^{(p,\theta}$ and Variable Lebesgue Spaces $L_{p(\cdot)}$, for a given set $\Omega \subset \mathbb{R}^n$, $|\Omega| = 1$, $1 < p < \infty$, and $\theta > 0$. The generalized Grand Lebesgue space $L^{p),\theta}(\Omega)$ consists of a measurable functions f such that

$$\|f\|_{p),\theta} = \sup_{0<\epsilon<p-1} \left(\epsilon^\theta \int_\Omega |f(x)|^{p-\epsilon}\, dx \right)^{\frac{1}{p-\epsilon}},$$

and the Small Lebesgue space $L^{(p,\theta}$ is defined as the associate space of $L^{p'),\theta}$, and so has the norm

$$\|f\|_{(p,\theta} = \sup \left\{ \int_\Omega f(x)g(x)\, dx : \|f\|_{p'),\theta} \leq 1 \right\}.$$

Particularly, we study conditions on the exponent function $p(\cdot)$ for there to be embeddings between the grand, small and variable Lebesgue spaces.

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