Multifractal analysis based on wavelet bases: Part 1. Mathematical foundations and p-leaders analysis.

Stéphane Jaffard
Université Paris Est Créteil, France
jaffard@u-pec.fr

Wavelet analysis allows to derive a classification of the pointwise singularities of an everywhere irregular signal f through a few exponents which describe the local behavior of the signal near the singularity. The associated multifractal spectrums measure the "size" of the sets with given exponents (via their Haudsdorff dimensions), and the multifractal formalism relates these dimensions with global regularity indices associated with the function spaces containing f. In applications, these indices are effectively computable from the wavelet expansion of the data, using an orthonormal wavelet basis through log-log plot regressions, and yield original classification and model selection tools. We will describe the mathematical foundations of this construction, with a specific focus on the Hölder exponent, and the p-exponent of Calderon and Zygmund. We will draw a comparison with alternative methods, such as the wavelet maxima method, or DFA (Detrended Fluctuation Analysis) We will derive the corresponding wavelet multiscale quantities, which are referred to, respectively, as wavelet leaders and p-leaders, and we will illustrate these techniques by several applications, ranging from fully developed turbulence, to stylometry (classification of Van Gogh pictures) or textometry (classification of literary texts).

Joint work with Patrice Abry (CNRS, ENS Lyon), Roberto Leonarduzzi (ENS Lyon and Hong-Kong University of Science and Technology), Clotilde Melot (Université de Marseille) and Herwig Wendt (CNRS, IRIT, Toulouse).