Measurable Steinhaus sets do not exist for finite sets or the integers in the plane

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A Steinhaus set S for a set B in Euclidean space is a set such that S has exactly one point in common with $t(B)$, for every rigid motion t of Euclidean space. We show here that if B is a finite set of at least two points then there is no such set S which is Lebesgue measurable. An old result of Komjath says that there exists a Steinhaus set for B being the set of integers on the x-axis in 2-space. We also show here that such a set cannot be Lebesgue measurable. We prove the latter result by way of showing that there is no measurable set in the plane which intersects almost every line L at measure 1 (this is still not possible if we ask that the intersection with almost every line is between two positive constants).

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