Dyadic harmonic analysis and weighted inequalities

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In this course we will introduce the basics of dyadic harmonic analysis and how it can be used to obtain weighted estimates for classical Calderón-Zygmund singular integral operators.

Harmonic analysts have used dyadic models for many years as a first step towards the understanding of more complex continuous operators. In 2000 Stephanie Petermichl discovered a representation formula for the venerable Hilbert transform as an average (over grids) of dyadic shift operators, allowing her to reduce arguments to finding (eg. sharp weighted) estimates for these simpler dyadic models. For the next decade the technique used to get sharp weighted inequalities was the Bellman function method introduced by Nazarov, Treil, and Volberg, paired with sharp extrapolation by Dragičević et al. Other methods where introduced by Hytönen, Lerner, Cruz-Uribe, Martell, Pérez, Lacey, Sawyer, Uriarte-Tuero, involving stopping time and median oscillation arguments, precursors of the very successful domination by positive sparse operators methodology. The culmination of this work was Hytönen's 2012 proof of the A_2 conjecture based on a representation formula for any Calderón-Zygmund operator as an average of appropriate dyadic operators. Since then domination by sparse dyadic operators has taken central stage and has found applications well beyond Hytönen's A_p-theorem.

We will discuss in some order:

1. Dyadic grids (random, adjacent) and Haar functions in $\mathbb{R}^d$ and briefly on spaces of homogeneous type.

2. Model dyadic operators including: dyadic maximal and square functions, martingale transform, Petermichl's Haar shifts, paraproducts, positive sparse operators.

3. Muckenhoupt A_p weights, Hytönen's A_p-theorem, sharp extrapolation.

4. Case study: the Hilbert transform and its commutator with a BMO function, one and two-weight inequalities.