What is variable bandwidth?

Karlheinz Gröchenig
Universität Wien, Austria
karlheinz.groechenig@univie.ac.at

We propose a new notion of variable bandwidth that is based on the spectral subspaces of an elliptic operator $A_p f = -\frac{d}{dx}(p(x)\frac{d}{dx})f$ where $p > 0$ is a strictly positive function. Denote by $c_\Lambda(A_p)$ the orthogonal ¿ projection of A_p corresponding to the spectrum of A_p in $\Lambda \subset \mathbb{R}^+$, the range of this projection is the space of functions of variable bandwidth with spectral set in Λ.

We develop the basic theory of these function spaces. First, we derive (nonuniform) sampling theorems, second, we prove necessary density conditions in the style of Landau. Roughly for a spectrum $\Lambda = [0, \Omega]$ the main results say that, in a neighborhood of $x \in \mathbb{R}$, a function of variable bandwidth behaves like a bandlimited function with local bandwidth $(\Omega/p(x))^{1/2}$.

Although the formulation of the results is deceptively similar to the corresponding results for classical bandlimited functions, the methods of proof are much more involved. On the one hand, we use the oscillation method from sampling theory and frame theoretic methods, on the other hand, we need the precise spectral theory of Sturm-Liouville operators and the scattering theory of one-dimensional Schrödinger operators.

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