Gabor Tight Fusion Frames: Construction and Applications in Phase Retrieval

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Fusion frame theory has recently garnered great interest among researchers who work in signal processing. Fusion frames extend the notion of a frame (i.e., an overcomplete set of vectors) within a Hilbert space $\mathcal{H}$ to a collection of subspaces $\{W_i\}_{i\in I}$ (with orthogonal projections $\{P_i\}_{i\in I}$) in $\mathcal{H}$. This concept was originally introduced by Kutyniok and Casazza in [3].

A tight fusion frame is one such that we have the identity $\sum_{i\in I} P_i = CI_{N\times N}$, ?i.e., the sum of the projections is a multiple of the identity. ?Such tight fusion frames are of interest for two reasons. First, they guarantee a very simple reconstruction of a signal; and second, tight fusion frames are robust against noise [2] and also remain robust against a one-erasure subspace when the rank of projections are equal to each other [5].

On the other hand, phaseless reconstruction is a field that has gathered interest in the mathematical community in the last decade. Phaseless reconstruction (or equivalently, phase retrieval) is defined as the recovery of a signal modulo phase from the absolute values of fusion frame measurement coefficients arising from a fusion frame. This is known to have applications to a disparate array of other scientific and applied disciplines, including X-ray crystallography [4], speech recognition [1, 6, 8], and quantum state tomography [7], where the recorded phase information of a signal is lost or distorted.

In the case of phase retrieval, the signal must be recovered from coefficients of dimension higher than one. Here, in the context of fusion frames, the problem is to recover $x \in \mathcal{H}_M$ "up to phase" from the measurements $\{\|P_i x\|\}_{i=1}^N$.

In this paper we demonstrate a new method for the construction of tight fusion frames. There are hithero few examples of tight fusion frames except trivial ones made up of orthogonal subspaces, so we believe this is a relevant and interesting advance. Moreover, there are few examples of phase retrieval fusion frames. In this paper, we present a condition that makes this structure allow phase retrieval.

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