Fourier decay of self-similar measures

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The decay properties of the Fourier transform $\widehat{\nu}(\xi)$ of a finite measure ν as $|\xi| \to \infty$ give crucial "arithmetic" information about ν. The Fourier transform $\widehat{\nu}(\xi)$ has *polynomial decay* if $|\widehat{\nu}(\xi)| \leq C|\xi|^{-\sigma/2}$ for some constants $C_\sigma, \sigma > 0$. The supreme of such σ is the *Fourier dimension* of ν.

The decay of Bernoulli convolutions (that is, self-similar measures for the iterated function systems $\{ax + t_i\}$ with weights p_i and $a \in (0,1)$) has been studied in classic works of Erdös [1,2] and Kahane [3].

In this context, in this work we present results about the polynomial decay of the Fourier transform outside a small set of exceptions, giving explicit estimates. Also, we study different types of dimensions of Bernoulli convolutions.

Bibliography:

[1] P.Erdös On a family of symmetric Bernoulli convolutions. Amer. J. Math., No. 61, 974–976, 1939.

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[3] J. P.Kahane Sur la distribution de certaines séries aléatoires. Bull. Soc. Math. France, No. 25, 119–122, 1971.

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