

BORDERLINE WEIGHTED ESTIMATES FOR THE MAXIMAL FUNCTION AND FOR ROUGH
SINGULAR INTEGRAL OPERATORS

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Muckenhoupt-Wheeden [2] in the seventies and Sawyer [5] in the eighties, established some one-dimensional highly nontrivial extensions of the weak type $(1, 1)$ property of the maximal function involving weights. These results were conjectured to hold for the Hilbert transform and for the maximal function in higher extensions. In the first part of this lecture we will survey about these conjectures that were proved and extended in different directions in [1] and [3]. Then we will discuss about the main open conjecture that has been recently settled in [4] for the maximal function, for Calderón-Zygmund operators (CZO) and for rough singular integrals. A key intermediate result is a classical L^p type estimate based on a good- λ inequality between CZO and the maximal function due to R. Coifman and C. Fefferman for A_∞ weights. We prove a corresponding results for rough singular integrals without having such a good- λ inequality. This result will be a consequence of a combination of an extrapolation theorem for the class A_∞ [7] combined with a sparse formula found by Conde-Culiuc-Di Plinio-Ou [8]. Two weighted results will be derived and further quantitative estimates will be discussed.

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