Regularity of refinable functions: matrix approach

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The compactly supported solution (refinable function) of the functional equation

$$\varphi(x) = \sum_{k \in \mathbb{Z}^s} c_k \varphi(Mx - k), \quad x \in \mathbb{R}^s, \quad c_k \in \mathbb{R}, \quad M \in \mathbb{Z}^{s \times s},$$

can generate systems of multivariate wavelets or frames. Refinable functions are building blocks for the limits of subdivision algorithms widely used in approximation and for generation of curves and surfaces. Refinable functions also naturally appear in applications in probability, number theory, and combinatorics.

In the univariate case, $M \geq 2$ is an integer, there are several efficient methods for determining the regularity of refinable functions. One of them, the so-called matrix approach, yields the Hoelder exponent of $\varphi \in C(\mathbb{R})$ and, in addition, provides a detailed analysis of the modulus of continuity of φ and of its local regularity. The generalization of the matrix approach to the multivariate case turned out to be a difficult task in the case of general dilation matrices M. The special case of isotropic dilation M (all eigenvalues of M are equal in the absolute value) is currently fully understood. In this talk we discuss the challenges of the anisotropic case, i.e. the dilation matrix M has eigenvalues different in the absolute value. We show how the Hoelder exponent of $\varphi \in C(\mathbb{R}^s)$ reflects the influence of the invariant subspaces of M corresponding to its different in modulus eigenvalues.

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