Frames and some algebraic forays

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We formulate a mathematical theory of frame multiplication, in which two essential algebraic operations can be made compatible in a natural way. The motivation comes from our approach to defining vector-valued ambiguity functions. These are formulated to provide realistic modelling of multi-sensor environments in which a useful time-frequency analysis is required, and they depend on the construction of finite number-theoretic sequences that have constant amplitude and zero-autocorrelation, i.e., CAZAC sequences.

The functions whose ambiguity function we wish to define have a given finite group G as their domain. Our results have the following form: i. if frame multiplication exists in the context of the aforementioned operations, then the vector-valued ambiguity function is well-defined; ii. frame multiplication exists if and only if the finite frames that arise in the theory are of a certain type, e.g., harmonic frames or, more generally, group frames.

For infinite locally compact abelian groups (LCAGs) G, we focus on those that are central in generalizations of p-adic number theory, i.e., G will have compact open subgroups H. We prove the analogue in this setting of the fact, true on Euclidean space and all similar LCAGs, in which frames of translates of f are characterized in terms of zero sets of periodizations of the Fourier transform of f. In the compact open subgroup case, there is a roadblock at the outset, and translation has to be correctly defined.

The frame multiplication theory is a collaboration with Travis Andrews and Jeffrey Donatelli. The earlier CAZAC work (2012), which is only a backdrop to introduce the setting of the talk, was a collaboration with Robert Benedetto and Joseph Woodward. The frames of translates result proved with Robert Benedetto, is essentially non-Euclidean in setting, but was guided by the analogous Euclidean theorem proved with Shidong Li in 1992.