Multivariate Self-similarity: Multiscale eigen structures for the estimation of Hurst exponents

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Scale invariance has become an ubiquitous paradigm massively used to model temporal dynamics in real-world data. Self-similar processes, and particularly their Gaussian instance, fractional Brownian motion, consist of the most common stochastic model used to account for scale invariance. However, most applications of self-similarity remained so far univariate, while data collected in real world applications most often naturally come as multivariate. Recently, Operator Fractional Brownian Motion (OfBm) has been proposed in the literature as the reference model for multivariate self-similarity. It yet remained barely used because of the lake of available identification procedure for the joint estimation of the parameters entering its definition. The present contribution achieves a first major step in the full identification of P-variate OfBm by proposing a procedure permitting to estimate the vector of P-Hurst exponents underlying its temporal dynamics. The proposed estimation procedure relies on the theoretical study of the multiscale eigen structure of the wavelet spectrum of OfBm. The proposed estimator is shown theoretically to be consistent, and to display asymptotic normality. Monte Carlo simulations applied to numerous independent copies of synthetic OfBm enable us to assess the estimation performance of the proposed procedure in a P-variate setting.

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