About Spetses

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Let $GL_n(q)$ be the group of invertible $n \times n$ matrices with entries in the finite field with q elements. Its order is the evaluation at $x = q$ of a polynomial in x which depends only on n. The orders of its "admissible subgroups" are as well evaluations at $x = q$ of polynomials in x, which divide the "order polynomial". The degrees of its unipotent characters are also evaluations at $x = q$ of polynomials in x, which divide the "order polynomial". There is a "Sylow theory" associated to each prime polynomial divisor of the order polynomial, and even modular representation theory may be described by polynomials evaluated at q. As if there were and object "$GL_n(x)$" which should specialize to $GL_n(q)$ for $x = q$.

The same phenomena occur for other finite groups of Lie type over finite fields, attached to Weyl groups of type B,D,E,F,G. As if there were groups of type B,D, etc, over a "field with x elements".

It turns out that, replacing the Weyl group by some complex reflection groups (the "special" ones), one can build up huge numerical and polynomial data which satisfy or generalize the data associated to the above groups. As if, this time, "groups over a field with x elements" do exist but cannot be specialized to actual finite groups by assigning a numerical value to x.

We shall try to talk about this program, initiated 23 years ago on the Greek Island named Spetses.

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