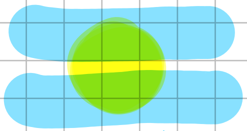

¡VAMOS ARGENTINAAA!

CLASE CONSULTAS MATE I B 2C 22

Ej 2

$$\begin{cases} 1+t = 2tx\,x' \\ x(1)=2 \end{cases}$$

$x' = \_\_\_\_$

Separación de variables

$$1+t = 2tx\,x'$$

$$\frac{1+t}{2t} = x\cdot\frac{dx}{dt}$$

$$\int \frac{1+t}{2t}\,dt = \int x\,dx$$

$$\int \frac{1}{2t} + \frac{1\cancel{t}}{2\cancel{t}}\,dt = \int x\,dx$$

$$\int \frac{1}{2}\cdot\frac{1}{t} + \frac{1}{2}\,dt = \int x\,dx$$

$$\frac{1}{2}\int \frac{1}{t}\,dt + \int \frac{1}{2}\,dt = \int x\,dx$$

$$\frac{1}{2}\cdot \ln|t| + \frac{1}{2}t + C = \frac{x^2}{2}$$

$$2\left(\frac{1}{2}\ln|t| + \frac{1}{2}t + c\right) = x^2$$

$$\sqrt{\ln|t| + t + 2C} = \sqrt{x^2}$$

$$\sqrt{\ln|t| + t + 2C} = |x|$$

$$x = \sqrt{\ln|t| + t + 2C} \quad (1)$$

$$x = -\sqrt{\ln|t| + t + 2C} \quad (2)$$

Dato inicial

(1) $x(1) = 2$

$$x(1) = \sqrt{\ln 1 + 1 + 2C} = 2$$

$$\sqrt{1+2C} = 2$$

$$1 + 2C = 4$$

$$2C = 3$$

$$\boxed{C = \frac{3}{2}}$$

Solución

$$\boxed{x(t) = \sqrt{\ln|t| + t + 3}}$$

(2) $x(1) = 2$

$$x(1) = -\sqrt{\ln 1 + 1 + 2C} = 2$$

$$= -\sqrt{1+2C} = 2$$

$$\sqrt{1+2C} = -2$$

El resultado de $\sqrt{\text{ALGO}} \geq 0$

No puedo despejar "c"

Consulta

Calcular la solución general del sistema:

$$\begin{cases} x' = -5x + 3y \\ y' = -6x + y \end{cases} \rightsquigarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = \underbrace{\begin{pmatrix} -5 & 3 \\ -6 & 1 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \end{pmatrix}$$

Buscamos autovalores y autovectores:

(I) $\begin{pmatrix} x \\ y \end{pmatrix} = \alpha e^{\lambda_1 t} (v_1) + \beta e^{\lambda_2 t} (v_2)$ si $\lambda_1, \lambda_2 \in \mathbb{R}$

(II) $\begin{pmatrix} x \\ y \end{pmatrix} = \alpha \, Re\left(e^{\lambda t} v_1\right) + \beta \, Im\left(e^{\lambda t} v_1\right)$ $\lambda, \bar{\lambda} \in \mathbb{C} \setminus \mathbb{R}$

$$\det\left[\begin{pmatrix} -5-\lambda & 3 \\ -6 & 1-\lambda \end{pmatrix}\right] = (-5-\lambda).(1-\lambda) + 18$$

$$\begin{aligned} \chi_A(\lambda) &= (-5-\lambda).(1-\lambda) + 18 \\ &= -5 + 4\lambda + \lambda^2 + 18 \\ &= \boxed{\lambda^2 + 4\lambda + 13} \end{aligned}$$

$\boxed{\lambda = -2+3i} \rightarrow V = \begin{pmatrix} 1-i \\ 2 \end{pmatrix}$

$\bar{\lambda} = -2-3i$

$$e^{\lambda t} v \rightarrow e^{(-2+3i)t} \begin{pmatrix} 1-i \\ 2 \end{pmatrix}$$

Recordar:

$$\boxed{e^{i\theta} = \cos(\theta) + i \,\text{sen}(\theta)}$$

$$e^{-2t} . e^{3it} \begin{pmatrix} 1-i \\ 2 \end{pmatrix}$$

$$e^{3it} = \cos(3t) + i \,\text{sen}(3t)$$

distributiva

$$e^{-2t} . \left(\cos(3t) + i \,\text{sen}(3t)\right) \begin{pmatrix} 1-i \\ 2 \end{pmatrix}$$

$$e^{-2t} . \begin{pmatrix} \cos(3t) - i\cos(3t) + i\,\text{sen}(3t) + \text{sen}(3t) \\ 2\cos(3t) + 2i\,\text{sen}(3t) \end{pmatrix}$$

$$\rightarrow \operatorname{Re}\left(e^{\lambda t} v\right) = e^{-2t} \cdot \begin{pmatrix} \cos(3t) + \operatorname{sen}(3t) \\ 2\cos(3t) \end{pmatrix}$$

$z = \underbrace{a}_{Re(z)} + \underbrace{b}_{Im(z)} i$

$$\rightarrow \underbrace{\operatorname{Im}\left(e^{\lambda t} v\right)}_{\text{¡es REAL!}} = e^{-2t} \cdot \begin{pmatrix} \overset{-\cos(3t) + \operatorname{sen}(3t)}{\operatorname{sen}(3t) - \cos(3t)} \\ 2\operatorname{sen}(3t) \end{pmatrix}$$

Todas las soluciones:

$$\boxed{\begin{pmatrix} x \\ y \end{pmatrix} = \alpha\, e^{-2t} \begin{pmatrix} \cos(3t) + \operatorname{sen}(3t) \\ 2\cos(3t) \end{pmatrix} + \beta\, e^{-2t} \begin{pmatrix} \operatorname{sen}(3t) - \cos(3t) \\ 2\operatorname{sen}(3t) \end{pmatrix} \qquad \alpha, \beta \in \mathbb{R}}$$

Consulta ¿cómo pensar las "condiciones iniciales"?

$$\begin{pmatrix} x \\ y \end{pmatrix} = \underbrace{\alpha\, e^{3t} (v_1)}_{\to +\infty} + \underbrace{\beta\, e^{-2t} (v_2)}_{\to 0} \qquad \text{si } \lambda_1, \lambda_2 \in \mathbb{R}$$

$$\lim_{t \to +\infty} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Necesito pedir $\alpha = 0$ para que $\displaystyle\lim_{t \to +\infty} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$