Equivariant K homology odd case

A G-C^* algebra

$\mathcal{E}_G^1(A) = \{(H, \psi, T, \pi)\}$

(H, ψ, π) is a covariant representation of A

$T \in \mathcal{L}(H)$

$$\left\{\begin{array}{ll} T = T^* & \\ \pi(g)T - T\pi(g) \in \mathcal{K}(H) & \forall\, g \in G \\ \psi(a)T - T\psi(a) \in \mathcal{K}(H) & \forall\, a \in A \\ \psi(a)(I - T^2) \in \mathcal{K}(H) & \forall\, a \in A \end{array}\right\}$$

$A, B \quad G\text{-}C^*$ algebras

$\varphi : A \to B \quad G$-equivariant $\quad *$-homomorphism

$\varphi^* : \mathcal{E}^1_G(B) \to \mathcal{E}^1_G(A)$

$$(H, \psi, T, \pi) \mapsto (H, \psi \circ \varphi, T, \pi)$$

$KK^1_G(A, \mathbb{C}) := \mathcal{E}^1_G(A)/\sim$

$\sim$ = "homotopy"

"homotopy" will be made precise later

addition in $KK^1_G(A, \mathbb{C})$ is direct sum

$$(H, \psi, T, \pi) + (H', \psi', T', \pi')$$
$$= (H \oplus H', \psi \oplus \psi', T \oplus T', \pi \oplus \pi')$$
$$-(H, \psi, T, \pi) = (H, \psi, -T, \pi)$$