Idea of the map

$$\mu : K_j^G(\underline{E}G) \longrightarrow K_j C_r^* G \qquad j = 0, 1$$

Assume $j = 0$ and G is compact

$$G \text{ compact} \implies \begin{cases} \underline{E}G = \bullet \\ K_0 C_r^* G = R(G) \end{cases}$$

$R(G)$ is the representation ring of G

$R(G)$ is the Grothendieck group of the category of finite dimensional (complex) representations of G

$R(G)$ is a free abelian group with one generator for each distinct (i.e. non-equivalent) irreducible representation of G

$\mu : KK_G^{\circ}(\mathbb{C},\mathbb{C}) \longrightarrow R(G)$

G compact $\Rightarrow$ Given $(H,\psi,T,\pi) \in \mathcal{E}_G^{\circ}(\mathbb{C})$ within the equivalence relation on $\mathcal{E}_G^{\circ}(\mathbb{C})$ we may assume

$$\psi(\lambda) = \lambda I \qquad \forall \lambda \in \mathbb{C}$$

$$T\pi(g) - \pi(g)T = 0 \qquad \forall g \in G$$

Hence the non-triviality of (H,ψ,T,π) is coming from

$$I - T^*T \in \mathcal{K}(H)$$

$$I - TT^* \in \mathcal{K}(H)$$

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These conditions $\Longrightarrow \begin{cases} \dim_{\mathbb{C}}(\text{Kernel } T) < \infty \\ \dim_{\mathbb{C}}(\text{Cokernel } T) < \infty \end{cases}$

Kernel (T) and Cokernel (T) are finite dimensional representations of G

$$\mu(H, \psi, T, \pi) = \text{Kernel}(T) - \text{Cokernel}(T)$$

$$\mu : KK_G^0(\mathbb{C}, \mathbb{C}) \longrightarrow K_0 C_r^* G = R(G)$$

Suppose G is non-compact

$$\mu : K_j^G(\underline{E}G) \longrightarrow K_j C_r^* G$$

The elements of $K_0^G(\underline{E}G)$ can be viewed as generalized elliptic operators on $\underline{E}G$. μ assigns to such a generalized elliptic operator its index.

$$\mu : K_0^G(\underline{E}G) \longrightarrow K_0 C_r^* G$$

$$\mu(H, \psi, T, \pi) = \mathrm{Kernel}(T) - \mathrm{CoKernel}(T)$$

THE CONJECTURE WITH COEFFICIENTS

Definition. A $G - C^*$ algebra is a C^* algebra A with a given continuous action of G.

$$G \times A \longrightarrow A$$

G acts on A by C^* algebra automorphisms.

The continuity condition is: For each $a \in A$,

$$G \longrightarrow A, g \mapsto ga$$

is a continuous map from G to A.

Let A be a $G-C^*$ algebra. Form the reduced crossed-product C^* algebra $C_r^*(G, A)$.

$K_j C_r^*(G, A) = ?$

$K_j^G(\underline{E}G, A)$ denotes the equivariant K-homology of $\underline{E}G$ with G-compact supports and coefficients A.

EXPANDER GRAPHS

A.LUBOTZKY and P.SARNAK

Let Γ be a finitely presented discrete group which contains an expander in its Cayley graph.

Such a Γ is a counter-example to the conjecture with coefficients.

Does such a Γ exist?

M. Gromov outlined a proof that such a Γ exists. A number of mathematicians are now filling in the details.

Conjecture(P.B. and A.Connes, 1980) Let G be a locally compact Hausdorff second countable topological group, and let A be any $G-C^*$ algebra, then

$$\mu : K_j^G(\underline{E}G, A) \longrightarrow K_j C_r^*(G, A)$$

is an isomorphism.

j = 0, 1.