$C_r^* G$ the reduced C* algebra of G

PROBLEM : $K_j C_r^* G = ?$ j = 0, 1

CONJECTURE (P.Baum - A. Connes) :

$\mu : K_j^G(\underline{E}G) \to K_j C_r^* G$ is an isomorphism.

j = 0, 1

$\underline{A}$ C^* ALGEBRA WITH UNIT 1_A

$GL(n, A)$ TOPOLOGICAL GROUP

$$GL(n, A) \hookrightarrow GL(n+1, A)$$

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \longmapsto \begin{bmatrix} a_{11} & \cdots & a_{1n} & 0 \\ \vdots & & \vdots & \vdots \\ a_{n1} & \cdots & a_{nn} & 0 \\ 0 & \cdots & 0 & 1_A \end{bmatrix}$$

$$GLA = \lim_{n \to \infty} GL(n, A)$$

DIRECT LIMIT TOPOLOGY

$$\underline{K_j A := \pi_{j-1}(GLA)}$$

$j = 1, 2, 3, \ldots$

$$\Omega^2 GLA \sim GLA$$

BOTT PERIODICITY

$$K_j A \cong K_{j+2} A$$

$j = 0, 1, 2, \ldots$

$K_0 A \quad K_1 A$

A C^* algebra with unit 1_A

$K_0 A = K_0^{alg}(A)$ = Grothendieck group of finitely generated (left) projective A-modules

$A = (A, \| \ \|, *)$

For $K_0 A$ can forget $\| \ \|$ and $*$

For $K_1 A$ cannot forget $\| \ \|$ and $*$

A C^* ALGEBRA

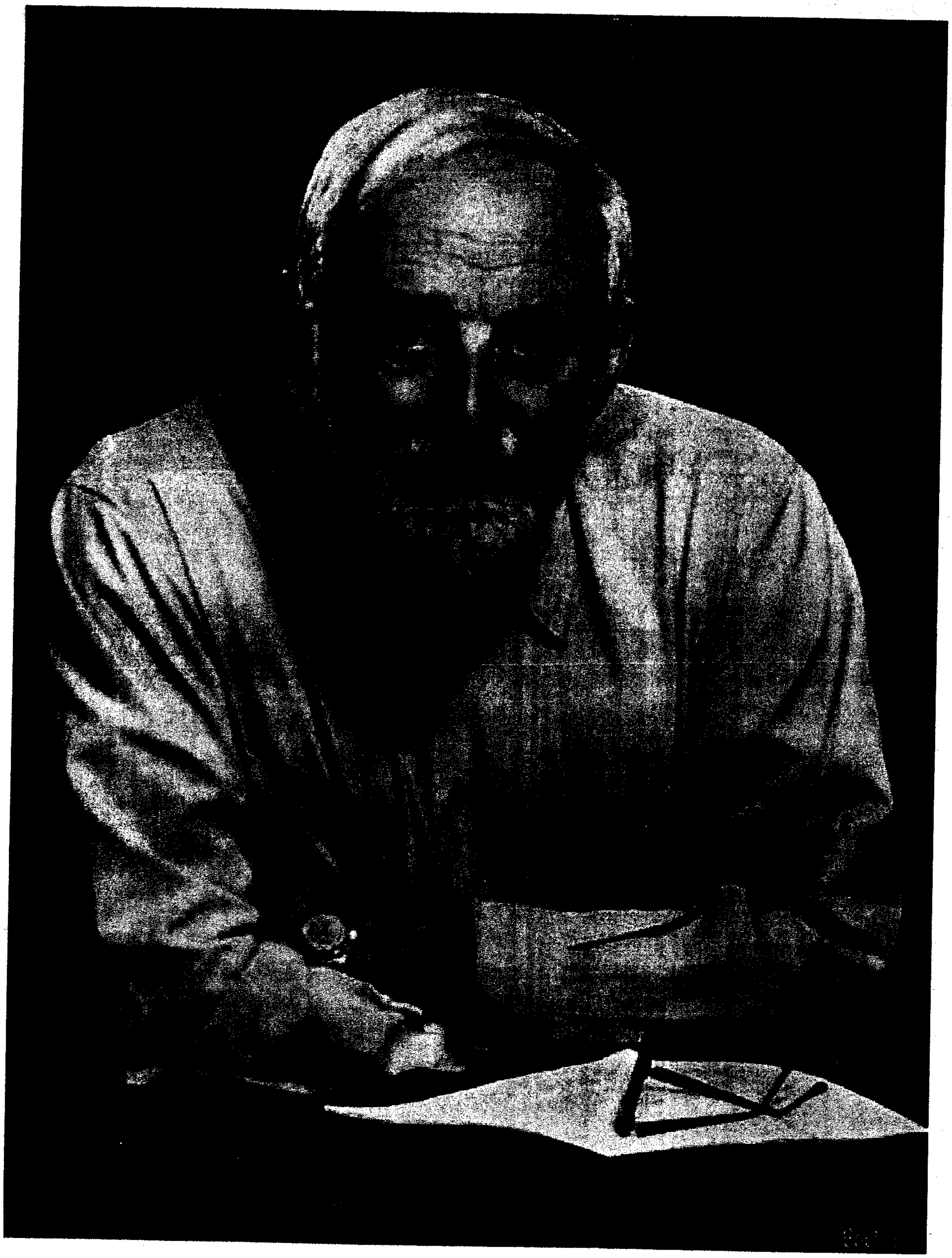
IF A IS NOT UNITAL, ADJOIN A UNIT

$$0 \longrightarrow A \longrightarrow \tilde{A} \longrightarrow \mathbb{C} \longrightarrow 0$$

DEFINE: $K_1 A = K_1 \tilde{A}$

$K_0 A = \text{Kernel}(K_0 \tilde{A} \longrightarrow K_0 \mathbb{C})$

$K_0 A \qquad K_1 A$



Raoul Bott

FUNCTORIALITY OF K-THEORY

A, B C^* algebras (with or without units)

$\varphi : A \longrightarrow B$ $\quad$ $*$-homomorphism

$\varphi_* : K_j A \longrightarrow K_j B$ $\quad$ homomorphism of abelian groups

$j = 0, 1$

$A \longmapsto K_j A$ is a functor

$(C^* \text{ algebras}) \longrightarrow (\text{abelian groups})$

THEOREM (R. BOTT)

$$\pi_j GL(n,\mathbb{C}) \quad = \quad \begin{cases} 0 & j \text{ even} \\ \mathbb{Z} & j \text{ odd} \end{cases}$$

$$j = 0, 1, 2, \ldots, 2n-1$$

$\mathbb{C}$ is a C^* algebra

$\| \lambda \| = |\lambda| \qquad \lambda \in \mathbb{C}$

$\lambda^* = \bar{\lambda}$

<u>**THEOREM**</u> (R. BOTT)

$$K_j \mathbb{C} = \begin{cases} \mathbb{Z} & j \text{ even} \\ 0 & j \text{ odd} \end{cases}$$

TOPOLOGICAL K-THEORY

X LOCALLY COMPACT HAUSDORFF TOPOLOGICAL SPACE

$K^0(X)$ $\qquad$ $K^1(X)$

DEFINED BY ATIYAH + HIRZEBRUCH

THIS IS TOPOLOGICAL K THEORY WITH COMPACT SUPPORTS

$$K^j(X) = K_j C_0(X) \qquad j = 0, 1$$

X COMPACT HAUSDORFF

$K^0(X) := $ GROTHENDIECK GROUP OF $\mathbb{C}$ VECTOR BUNDLES ON X

X locally compact Hausdorff topological space

$$ch : K^j(X) \to \bigoplus_l H_c^{j+2l}(X;\mathbb{Q}) \qquad j = 0\,,\,1$$

<u>Theorem</u>. For any locally compact Hausdorff topological space X

$$ch : K^j(X) \to \bigoplus_l H_c^{j+2l}(X;\mathbb{Q}) \qquad j = 0,1$$

is a rational isomorphism, i.e.

$$K^j(X) \otimes_{\mathbb{Z}} \mathbb{Q} \to \bigoplus_l H_c^{j+2l}(X;\mathbb{Q})$$

is an isomorphism. j = 0,1

Cech cohomology (with compact supports)
Alexander-Spanier (with compact supports)
Representable (with compact supports)

can be used.
