# Calculo de integrales de una variable

Calcular las siguientes integrales :

1. $\displaystyle\int \left(x^{3/2} + \frac{\sqrt{x}}{5} - \frac{2}{x}\right) dx$
2. $\displaystyle\int x^3\sqrt{x^4+5}\,dx$
3. $\displaystyle\int (\cos(2x) - 2\sin(x))\,dx$
4. $\displaystyle\int 2^x\,dx$
5. $\displaystyle\int \cos(x)e^{\sin(x)}\,dx$
6. $\displaystyle\int x\sin(x^2)\,dx$
7. $\displaystyle\int \cos(x)\sin(x)^3\,dx$
8. $\displaystyle\int \frac{dx}{(1+x)(2+x)}$
9. $\displaystyle\int \frac{3x+7}{x^2+6x+8}\,dx$
10. $\displaystyle\int \frac{e^t}{1+e^t}\,dt$
11. $\displaystyle\int \frac{(\ln x)^2}{x}\,dx$
12. $\displaystyle\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$
13. $\displaystyle\int \sqrt{1+\sqrt{x}}\,dx$
14. $\displaystyle\int \frac{dx}{x^2+6x+14}$
15. $\displaystyle\int \frac{x^2}{x^2+4}\,dx$
16. $\displaystyle\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx$
17. $\displaystyle\int_0^{\pi/4} \frac{\tan^3\theta}{\cos^2\theta}\,d\theta$
18. $\displaystyle\int x^2\sin(4x)\,dx$
19. $\displaystyle\int (x^2+1)\ln(x)\,dx$
20. $\displaystyle\int \sin^4(x)\,dx$

## Corrección

1. $\displaystyle\int \left(x^{3/2} + \frac{\sqrt{x}}{5} - \frac{2}{x}\right) dx = \frac{2}{5}x^{5/2} + \frac{2}{15}x^{3/2} - 2\ln|x| + C$
2. con $u(x) = x^4+5$,
$$\int x^3\sqrt{x^4+5}\,dx = \frac{1}{4}\int u'(x)\sqrt{u(x)}\,dx = \frac{1}{6}\int (u(x)^{3/2})'\,dx = \frac{1}{6}u(x)^{3/2} + C = \frac{1}{6}(x^4+5)^{3/2} + C$$
3. $\displaystyle\int (\cos(2x) - 2\sin(x))\,dx = \frac{1}{2}\sin(2x) + 2\cos(x) + C$
4. $\displaystyle\int 2^x\,dx = \int e^{\ln(2)x}\,dx = \frac{1}{\ln(2)}\int \ln(2)e^{\ln(2)x}\,dx = \frac{1}{\ln(2)}e^{\ln(2)x} + C = \frac{1}{\ln(2)}2^x + C$
5. $\displaystyle\int \cos(x)e^{\sin(x)}\,dx = \int u'(x)e^{u(x)}\,dx = e^{u(x)} + C = e^{\sin(x)} + C$, con $u(x) = \sin(x)$
6. $\displaystyle\int x\sin(x^2)\,dx = \frac{1}{2}\int u'(x)\sin(u(x))\,dx = -\frac{1}{2}\cos(u(x)) + C = -\frac{1}{2}\cos(x^2) + C$, con $u(x) = x^2$

7. $\int \cos(x)\sin(x)^3\,dx = \int u'(x)u(x)^3\,dx = \frac{1}{4}u(x)^4 = \frac{1}{4}\sin(x)^4 + C$

8. $\int \frac{dx}{(1+x)(2+x)} = \int \left(\frac{1}{1+x} - \frac{1}{2+x}\right)dx = \ln|1+x| - \ln|x+2| + C$

9. $\int \frac{3x+7}{x^2+6x+8}\,dx = \frac{1}{2}\int \frac{dx}{x+2} + \frac{5}{2}\int \frac{dx}{x+4} = \frac{1}{2}\ln|x+2| + \frac{5}{2}\ln|x+4| + C$

10. haciendo la substitución $x = e^t$, $dx = e^t dt = x dt$, obtenemos

$$\int \frac{e^t}{1+e^t}\,dt = \int \frac{dx}{1+x} = \ln|1+x| + C = \ln|1+e^t| + C = \ln(1+e^t) + C$$

11. haciendo la substitución $y = ln(x)$, $dy = dx/x$, obtenemos

$$\int \frac{(\ln x)^2}{x}\,dx = \int y^2 dy = \frac{1}{3}y^3 + C = \frac{1}{3}(\ln x)^3 + C$$

12. haciendo la substitución $y = \sqrt{x}$, $dy = \frac{dx}{2\sqrt{x}} \Leftrightarrow dx = 2y dy$ , obtenemos

$$\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx = 2\int \cos(y)\,dy = 2\sin(y) + C = 2\sin(\sqrt{x}) + C$$

13. haciendo la substitución $y = 1+\sqrt{x}$, $dy = \frac{dx}{2\sqrt{x}} \Leftrightarrow dx = 2(y-1)dy$ , obtenemos

$$\begin{aligned}\int \sqrt{1+\sqrt{x}}\,dx &= 2\int y^{1/2}(y-1)\,dy = 2\int (y^{3/2} - y^{1/2})\,dy = \frac{4}{5}y^{5/2} - \frac{4}{3}y^{3/2} + C \\ &= \frac{4}{5}(1+\sqrt{x})^{5/2} - \frac{4}{3}(1+\sqrt{x})^{3/2} + C\end{aligned}$$

14. haciendo la substitución $y = \frac{x+3}{\sqrt{5}}$, $dy = dx/\sqrt{5}$, obtenemos

$$\begin{aligned}\int \frac{dx}{x^2+6x+14} &= \int \frac{dx}{(x+3)^2+5} = \frac{1}{5}\int \frac{dx}{\left(\frac{x+3}{\sqrt{5}}\right)^2 + 1} = \frac{1}{\sqrt{5}}\int \frac{dy}{y^2+1} = \frac{1}{\sqrt{5}}\arctan(y) + C \\ &= \frac{1}{\sqrt{5}}\arctan\left(\frac{x+3}{\sqrt{5}}\right) + C\end{aligned}$$

15. haciendo la substitución $y = \frac{x}{2}$, obtenemos

$$\begin{aligned}\int \frac{x^2}{x^2+4}\,dx &= \int \frac{x^2+4-4}{x^2+4}\,dx = x - 4\int \frac{dx}{x^2+4} + C = x - \int \frac{dx}{\left(\frac{x}{2}\right)^2+1} + C \\ &= x - 2\int \frac{dy}{y^2+1} + C = x - 2\arctan(y) + C = \\ &= x - 2\arctan\left(\frac{x}{2}\right) + C\end{aligned}$$

16. haciendo la substitución $y = e^{-2x}$, $dy = -2y dx$, obtenemos

$$\begin{aligned}\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx &= \int \frac{1-e^{-2x}}{1+e^{-2x}}\,dx = \frac{1}{2}\int \frac{y-1}{y(1+y)}\,dy = \int \frac{dy}{1+y} - \frac{1}{2}\int \frac{dy}{y} \\ &= \ln(1+e^{-2x}) + x + C\end{aligned}$$

17. haciendo la substitución $y = \tan(\theta)$, $dy = \frac{d\theta}{\cos^2(\theta)}$, obtenemos

$$\int_0^{\pi/4} \frac{\tan^3\theta}{\cos^2\theta}\,d\theta = \int_0^{\pi/4} y^3\,dy = \frac{1}{4}y^4\left(\frac{\pi}{4}\right) = \frac{1}{4}$$

pues $\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)$ $(= \sqrt{2}/2)$.

18. haciendo dos integración por parte succesivamente (con $\int \sin(4x)\,dx = -\frac{1}{4}\cos(4x)$), obtenenmos

$$\begin{aligned}
\int x^2 \sin(4x)\,dx &= -\frac{1}{4}x^2\cos(4x) - \int (2x)\left(-\frac{1}{4}\cos(4x)\right)\,dx + C = -\frac{1}{4}x^2\cos(4x) + \frac{1}{2}\int x\cos(4x)\,dx + C \\
&= -\frac{1}{4}x^2\cos(4x) + \frac{1}{8}x\sin(4x) - \frac{1}{8}\int \sin(4x)\,dx + C \\
&= -\frac{1}{4}x^2\cos(4x) + \frac{1}{8}x\sin(4x) + \frac{1}{32}\cos(4x) + C
\end{aligned}$$

19.

$$\begin{aligned}
\int (x^2+1)\ln(x)\,dx &= \left(\frac{1}{3}x^3 + x\right)\ln(x) - \int \left(\frac{1}{3}x^3 + x\right) \times \frac{1}{x}\,dx + C \\
&= \left(\frac{1}{3}x^3 + x\right)\ln(x) - \int \left(\frac{1}{3}x^2 + 1\right) + C \\
&= \left(\frac{1}{3}x^3 + x\right)\ln(x) - \frac{1}{9}x^3 - x + C
\end{aligned}$$

20. como $\sin^2(x) = 1 - \cos^2(x)$ por todo $x \in \mathbb{R}$,

$$\begin{aligned}
\int \sin^4(x)\,dx = \int \sin^3(x)\sin(x)\,dx &= -\sin^3(x)\cos(x) + 3\int \cos^2(x)\sin^2(x)\,dx \\
&= -\sin^3(x)\cos(x) + 3\int \sin^2(x)\,dx - 3\int \sin^4(x)\,dx
\end{aligned}$$

entonces

$$\int \sin^4(x)\,dx = -\frac{1}{4}\sin^3(x)\cos(x) + \frac{3}{4}\int \sin^2(x)\,dx$$