Hyperelliptic d-tangential covers and $d \times d$-matrix KdV elliptic solitons

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Abstract

More than 40 years ago I. Krichever developped the Theory of (vector) B-A functions and devised a criterion for a d-marked compact Riemann surface to provide $d \times d$-matrix solutions to the KdV equation. Later on he also found a criterion for a d-marked curve to provide $d \times d$-matrix solutions to the KP equation, doubly periodic with respect to x, the first KP flow. In particular, when both criteria apply one should obtain $d \times d$-matrix KdV elliptic solitons. It seems however that the latter issue has been completely neglected until very recently (cf.: [10] where the $d = 2$ case is treated). In this article we fix a complex elliptic curve $X = \mathbb{C}/\Lambda$, corresponding to a lattice $\Lambda \subset \mathbb{C}$, and define so-called hyperelliptic d-tangential covers as d-marked covers of X satisfying a geometric condition inside their Jacobians. They satisfy Krichever's criteria and give rise, therefore, to families of $d \times d$-matrix KdV elliptic solitons. We also construct an anticanonical rational surface $\mathcal{S}$ naturally attached to X, with a Picard group of rank 10. It turns out that the former covers of X correspond to rational irreducible curves in suitable divisor classes of $\mathcal{S}$. We thus reduce their construction to proving that the associated Severi Varieties (of rational irreducible nodal curves) are not empty. The final key to the problem consists in finding rational reducible nodal curves in the latter divisor classes which can be deformed to irreducible ones, according to A. Tannenbaum's criterion (see [5]). At last we deduce, for any $d \geq 2$, infinite families (of arbitrary high genus and degree) of hyperelliptic d-tangential covers, giving rise to $d \times d$-matrix KdV elliptic solitons.