ADI SPLITTING SCHEMES FOR $TV - H^{-1}$ INPAINTING.

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Abstract: The general idea of every *splitting method* is breaking down a complicated problem into smaller (and, typically, easier to approach) parts, such that these smaller problems can be solved efficiently. *Alternating direction implicit* (ADI) methods are an example of dimensional splitting, i.e. methods that reduce the computations related to the problem to become actually one-dimensional. We apply such a scheme to the partial differential equation related to the problem of *inpainting* in image processing, where the goal is filling the missing parts of damaged images using the information obtained from the surrounding areas. As the standard second order approaches present some drawbacks in connecting edges over large distances and in the smooth propagation of level lines into the damaged domain, we focus on a higher order method, the $TV - H^{-1}$ model where the regularizing term is the laplacian of elements belonging to the subdifferential of the total variation of the image. Such methods have been recently studied showing better and more accurate results, but imposing, from a numerical point of view, some prohibitive restriction on the time-step size Δt with respect to the spatial one Δx of the order $\Delta t = O(\Delta x)^4$. A careful application of the ADI methods seem to overcome this limit, thus allowing larger time-steps and efficient numerical solutions but imposing some conditions on the parameter regularizing the total variation element.