A free boundary problem with gradient constraint and Tug-of-War games

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We deal with the asymptotic behaviour (as $p \to \infty$) of solutions for elliptic free boundary problems of $p-$Laplacian type ($2 \leq p < \infty$):

$$-\Delta_p u(x) + \lambda_0(x)\chi_{\{u>0\}}(x) = 0 \quad \text{in} \quad \Omega \subset \mathbb{R}^N, \tag{0.1}$$

with a prescribed Dirichlet boundary data, where $\lambda_0 > 0$ is a bounded function and Ω is a regular domain. First, we prove the convergence as $p \to \infty$ of any family of solutions $(u_p)_{p \geq 2}$, as well as we obtain the corresponding limit operator (in non-divergence form) ruling the limit equation,

$$\begin{cases} \max\left\{-\Delta_\infty u_\infty, \; -|\nabla u_\infty| + \chi_{\{u_\infty > 0\}}\right\} & = & 0 & \text{in} & \Omega \cap \{u_\infty \geq 0\} \\ u_\infty & = & g & \text{on} & \partial\Omega. \end{cases}$$

Next, we obtain uniqueness for solutions to this limit problem together with a number of weak geometric and measure theoretical properties as non-degeneracy, uniform positive density, porosity and convergence of the free boundaries.

Finally, we show that any solution to the limit operator is a limit of value functions for a specific Tug-of-War game.

Joint work with P. Blanc[1] and J.V. da Silva.[2]

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