Elliptic fibrations and K3 surfaces

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Plan

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- Introduction
- Problem
- Method
- Classification

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Curves

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Special attention to : Elliptic Curves !

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Our goal today : Look for elliptic curves inside a K3 surfaces.

Generalizing elliptic curves/ The surfaces we consider

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A K3 surface is a smooth algebraic surface such that :

- $K_X \equiv 0$
- $h^1(X, \mathcal{O}_X) = 0$ *(regular).*

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Tool/Method : Lattice structure of the Néron-Severi and Mordell-Weil groups.

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- Bertin-Lecacheux : Modular K3 associated to $\Gamma_1(8)$.
- BGHLMSW (Women in Numbers Europe) : A singular K3 with transcendental lattice isometric to $< 2 > \oplus < 6 >$.

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Idea : Ask what happens to elliptic fibrations on X when they descend to X/ι ?

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Today : To avoid case *iii*), assume ι^* acts as the identity on $\mathrm{NS}(X)$.

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Task : Classify all conic bundles on X/ι.

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Consider X a $K3$ surface with a non-symplectic involution ι "acting" as the identity on $\mathrm{NS}(X)$.

Lemma

Let $\pi : X \to \mathbb{P}^1$ be an elliptic fibration induced by a conic bundle. Then $\mathrm{MW}(\pi) \subseteq \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$. Moreover, ι it fixes a curve which is not rational then $\mathrm{MW}(\pi) \subseteq \mathbb{Z}/2\mathbb{Z}$.

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$k = 9$:

trivial lattice	$MW(\mathcal{E})$
$U \oplus A_8$	$\mathbb{Z}/3\mathbb{Z}$
$U \oplus E_8 \oplus E_8 \oplus A_1$	$\{1\}$
$U \oplus E_7 \oplus D_{10}$	$\mathbb{Z}/2\mathbb{Z}$
$U \oplus D_{16} \oplus A_1$	$\mathbb{Z}/2\mathbb{Z}$

(1)

$k = 8:$

trivial lattice	$MW(\mathcal{E})$
$U \oplus A_7$	$\mathbb{Z}$
$U \oplus E_8 \oplus E_7 \oplus A_1$	$\{1\}$
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$U \oplus E_7 \oplus E_7 \oplus A_1 \oplus A_1$	$\mathbb{Z}/2\mathbb{Z}$
$U \oplus E_7 \oplus D_8 \oplus A_1$	$\mathbb{Z}/2\mathbb{Z}$
$U \oplus D_{16}$	$\{1\}$
$U \oplus D_{14} \oplus A_1 \oplus A_1$	$\mathbb{Z}/2\mathbb{Z}$
$U \oplus D_{12} \oplus D_4$	$\mathbb{Z}/2\mathbb{Z}$
$U \oplus D_{10} \oplus D_6$	$\mathbb{Z}/2\mathbb{Z}$
$U \oplus D_8 \oplus D_8$	$\mathbb{Z}/2\mathbb{Z}$

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$k = 7:$

trivial lattice	$MW(\mathcal{E})$
$U \oplus A_7$	$\mathbb{Z}^2$
$U \oplus E_8 \oplus D_6 \oplus A_1$	$\{1\}$
$U \oplus E_7 \oplus D_6 \oplus A_1 \oplus A_1$	$\mathbb{Z}/2\mathbb{Z}$
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trivial lattice	$MW(\mathcal{E})$
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$U \oplus E_8 \oplus D_4 \oplus A_1 \oplus A_1$	$\{1\}$
$U \oplus E_7 \oplus D_6 \oplus A_1$	$\{1\}$
$U \oplus E_7 \oplus D_4 \oplus A_1 \oplus A_1 \oplus A_1$	$\mathbb{Z}/2\mathbb{Z}$
$U \oplus D_{12} \oplus A_1^2$	$\{1\}$
$U \oplus D_{10} \oplus D_4$	$\{1\}$
$U \oplus D_{10} \oplus A_1^4$	$\mathbb{Z}/2\mathbb{Z}$
$U \oplus D_8 \oplus D_6$	$\{1\}$
$U \oplus D_8 \oplus D_4 \oplus A_1 \oplus A_1$	$\mathbb{Z}/2\mathbb{Z}$
$U \oplus D_6 \oplus D_6 \oplus A_1 \oplus A_1$	$\mathbb{Z}/2\mathbb{Z}$

(4)

$k = 5:$

trivial lattice	$MW(\mathcal{E})$
$U \oplus A_5$	$\mathbb{Z}^4$
$U \oplus E_8 \oplus A_1^5$	$\{1\}$
$U \oplus E_7 \oplus D_4 \oplus A_1 \oplus A_1$	$\{1\}$
$U \oplus E_7 \oplus A_1^6$	$\mathbb{Z}/2\mathbb{Z}$
$U \oplus D_{10} \oplus A_1 \oplus A_1 \oplus A_1$	$\{1\}$
$U \oplus D_8 \oplus D_4 \oplus A_1$	$\{1\}$
$U \oplus D_8 \oplus A_1^5$	$\mathbb{Z}/2\mathbb{Z}$
$U \oplus D_6 \oplus D_6 \oplus A_1$	$\{1\}$
$U \oplus D_6 \oplus D_4 \oplus A_1^3$	$\mathbb{Z}/2\mathbb{Z}$

(5)

$k = 4:$

trivial lattice	$MW(\mathcal{E})$
$U \oplus A_4$	$\mathbb{Z}^5$
$U \oplus E_7 \oplus A_1^5$	$\{1\}$
$U \oplus D_8 \oplus A_1^4$	$\{1\}$
$U \oplus D_6 \oplus D_4 \oplus A_1^2$	$\{1\}$
$U \oplus D_6 \oplus A_1^6$	$\mathbb{Z}/2\mathbb{Z}$
$U \oplus D_6 \oplus A_1^6$	$\mathbb{Z}/2\mathbb{Z}$
$U \oplus D_4 \oplus D_4 \oplus D_4$	$\{1\}$
$U \oplus D_4 \oplus D_4 \oplus A_1^4$	$\mathbb{Z}/2\mathbb{Z}$

(6)

$k = 3:$

trivial lattice	$MW(\mathcal{E})$
$U \oplus A_3$	$\mathbb{Z}^6$
$U \oplus D_6 \oplus A_1^5$	$\{1\}$
$U \oplus D_4 \oplus D_4 \oplus A_1^3$	$\{1\}$
$U \oplus D_4 \oplus A_1^7$	$\mathbb{Z}/2\mathbb{Z}$

(7)

$k = 2:$

trivial lattice	$MW(\mathcal{E})$
$U \oplus A_2$	$\mathbb{Z}^7$
$U \oplus D_4 \oplus A_1^6$	$\{1\}$
$U \oplus A_1^{10}$	$\mathbb{Z}/2\mathbb{Z}$

(8)

$k = 1:$

trivial lattice	$MW(\mathcal{E})$
$U \oplus A_1$	$\mathbb{Z}^8$
$U \oplus A_1^9$	$\{1\}$

(9)

Obrigada ! Muchas Gracias !