dem Teorema 1

Vimos que $T_p^2 - T_{p1}^2 = y_{2.1}^T \, W_{22.1}^{-1} \, y_{2.1}$

donde $y_{2.1} = y^{(2)} - B y^{(1)}$ $\quad B = W_{21} W_{11}^{-1}$

(a) Probaremos que

$$y_{2.1} \Big|_{y^{(1)}, U} \sim N_{p-p_1}\left(\mu_{2.1}, \Sigma_{22.1}\left(1 + \frac{T_{p1}^2}{n}\right)\right)$$

Sabemos que

$$y^{(2)} \Big|_{y^{(1)}} \sim N_{p-p_1}\left(\mu_{2.1} + \beta y^{(1)}, \Sigma_{22.1}\right)$$

Además como $X$ es indep de $W$ $\quad y^{(2)}$ es indep de $W$ y de $V$.

$$y_{2.1} = y^{(2)} - B y^{(1)} = y^{(2)} - W_{21} W_{11}^{-1} y^{(1)}$$

$$= y^{(2)} - V^T U (U^T U)^{-1} y^{(1)}$$

Sea $a = U (U^T U)^{-1} y^{(1)}$ luego

$$y_{2.1} = y^{(2)} - V^T a$$

Como $y$ es indep de $W$, $y^{(2)} \perp V$

$$V^T a = \sum_{i=1}^{m} v_i a_i \quad \therefore$$

$$y^{(2)} \Big|_{U, y^{(1)}} \perp V^T a \Big|_{U, y^{(1)}}$$

$V^T a \Big|_{U, x^{(1)}}$ ~~[illegible]~~ es normal

$$\boxed{\begin{aligned} &E v_i|_{u_i} = \Sigma_{21} \Sigma_{11}^{-1} u_i \\ &V^T = (v_1 \dots v_n) \end{aligned}}$$

luego

$y_{2.1} \Big|_{U, y^{(1)}}$ es normal

$$E\left(y_{2.1} \Big| U, y^{(1)}\right) = E\left(y^{(2)} \Big| y^{(1)}\right) - E\left[V^T \Big| U\right] a$$

$$= \mu_{2.1} + \beta y^{(1)} - \Sigma_{21} \Sigma_{11}^{-1} U^T a$$

$$= \mu_{2.1} + \beta y^{(1)} - \underbrace{\Sigma_{21} \Sigma_{11}^{-1}}_{\beta} \underbrace{U^T U (U^T U)^{-1}}_{I} y^{(1)}$$

$$= \mu_{2.1}$$

$$\boxed{E \, y_{2.1} \Big|_{U, y^{(1)}} = \mu_{2.1}}$$

$$\mathrm{Cov}\left(y_{2.1} \Big|_{U, y^{(1)}}\right) = \mathrm{Cov}\left(y^{(2)} \Big|_{U, y^{(1)}}\right) + \mathrm{Cov}\left(V^T a \Big|_{y^{(1)}, U}\right)$$

$$= \Sigma_{22.1} + \|a\|^2 \Sigma_{22.1}$$

$$= (1 + \|a\|^2) \Sigma_{22.1}$$

Pero

$$\|a\|^2 = y^{(1)T} (U^T U)^{-1} \cancel{U^T U} \cancel{(U^T U)^{-1}} y^{(1)}$$

$$= y^{(1)T} W_{11}^{-1} y^{(1)} = T_{p1}^2 / n$$

(b) $y_{2\cdot 1}$ es indep de $W_{22\cdot 1}$

$W_{22\cdot 1}$ es indep de $(W_{11}, W_{12})$
o sea $(U, B)$
además es indep de $y$ luego
$W_{22\cdot 1}$ es indep de $y_{2\cdot 1}$ q'es $f^n$ de $(y, B)$

$$W_{22\cdot 1}\Big|_{y^{(1)}, W_{11}} \sim W(\Sigma_{22\cdot 1}, p-p_1, m-p_1)$$

indep

$$y_{2\cdot 1}\Big|_{y^{(1)}, W_{11}} \sim N\left(\mu_{2\cdot 1}, \Sigma_{22\cdot 1}\left(1+\frac{T^2_{p_1,m}}{m}\right)\right)$$

$\Rightarrow$

$$x = \left(1+\frac{T^2_{p_1,m}}{m}\right)^{-1/2} y_{2\cdot 1}\Big|_{y^{(1)}, W_{11}} \sim N_{p-p_1}(\nu, \Sigma_{22\cdot 1})$$

con $\nu = \dfrac{\mu_{2\cdot 1}}{\sqrt{1+\frac{T^2_{p_1,m}}{m}}}$

$\Rightarrow (m-p_1)\, x^T W_{22\cdot 1}^{-1} x\Big|_{y^{(1)}, W_{11}}$ es un Hotelling no central

o sea

$$(m-p_1)\frac{y_{2\cdot 1}^T W_{22\cdot 1} y_{2\cdot 1}}{\left(1+\frac{T^2_{p_1,m}}{m}\right)}\Bigg|_{y^{(1)}, W_{11}}$$

$\sim$ Hotelling no central con parametro

$$\lambda^2 = \nu^T \Sigma_{22\cdot 1}\nu = \frac{\mu_{2\cdot 1}^T \Sigma_{22\cdot 1}\mu_{2\cdot 1}}{1+\frac{T^2_{p_1,m}}{m}}$$

es decir

$$(m-p_1)\frac{y_{2\cdot 1}^T W_{22\cdot 1} y_{2\cdot 1}}{\left(1+\frac{T^2_{p_1,m}}{m}\right)}\frac{\overbrace{(m-p_1-(p-p_1)+1)}^{m-p+1}}{(m-p_1)(p-p_1)}$$

$$\sim F_{p-p_1, m-p+1}(\lambda^2)$$

Pero $y_{2\cdot 1}^T W_{22\cdot 1} y_{2\cdot 1} = T_p^2 - T_{p_1}^2$

de donde el resultado