Datos (en cada nivel del factor)

$$\begin{aligned} &Y_{1,1}, Y_{1,2}, \ldots, Y_{1,n_1}, \\ &Y_{2,1}, Y_{2,2}, \ldots, Y_{2,n_2} \\ &\quad \vdots \\ &Y_{k,1}, Y_{k,2}, \ldots, Y_{k,n_k} \end{aligned}$$

Sean

$$n = \sum_{i=1}^{k} n_i, \qquad \bar{Y}_{i\bullet} = \frac{1}{n_i} \sum_{j=1}^{n_i} Y_{ij}$$

$$\bar{Y}_{\bullet\bullet} = \frac{1}{n} \sum_{i=1}^{k} \sum_{j=1}^{n_i} Y_{ij}, \qquad S_i^2 = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{i\bullet})^2$$

Suma de cuadrados *Between* (entre niveles) $$SS_B = \sum_{i=1}^{k} n_i (\bar{Y}_{i\bullet} - \bar{Y}_{\bullet\bullet})^2$$

Suma de cuadrados *Within* (dentro) $$SS_W = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{i\bullet})^2 = \sum_{i=1}^{k} (n_i - 1) S_i^2$$

Suma de cuadrados total, $$SS_T = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{\bullet\bullet})^2$$

Vale la identidad, $SS_T = SS_B + SS_W$

Distribuciones muestrales: bajo los supuestos del modelo ANOVA, y bajo H_0,

$$\frac{SS_B}{\sigma^2} \sim \chi^2_{k-1}, \quad \frac{SS_W}{\sigma^2} \sim \chi^2_{n-k}, \quad \text{independientes}, \quad F = \frac{SS_B/(k-1)}{SS_W/(n-k)} \sim F_{k-1,n-k}$$

Comparaciones múltiples: bajo los supuestos del modelo ANOVA

- Sea $S_p^2 = \frac{SS_W}{n-k}$, tenemos que

$$\frac{(\bar{Y}_{h\bullet} - \mu_h) - (\bar{Y}_{g\bullet} - \mu_g)}{\sqrt{\sigma^2 \left(\frac{1}{n_h} + \frac{1}{n_g}\right)}} \sim N(0,1)$$

$$\frac{SS_W}{\sigma^2} = \frac{\sum_{i=1}^{k} (n_i - 1) S_i^2}{\sigma^2} \sim \chi^2_{n-k}$$

$$\frac{(\bar{Y}_{h\bullet} - \mu_h) - (\bar{Y}_{g\bullet} - \mu_g)}{S_p \sqrt{\frac{1}{n_h} + \frac{1}{n_g}}} \sim t_{n-k}$$

- Si $n_i = J$ para todo i, tenemos que

$$\max_{h,g} \sqrt{2} \frac{|(\bar{Y}_{h\bullet} - \mu_i) - (\bar{Y}_{g\bullet} - \mu_j)|}{S_p \sqrt{\frac{1}{n_h} + \frac{1}{n_g}}} \sim q_{k,n-k}$$

Intervalos de Confianza simultáneos: m comparaciones

- Bonferroni: $\bar{Y}_{h\bullet} - \bar{Y}_{g\bullet} \quad \pm \quad t_{n-k,(\alpha/2m)} S_p \sqrt{\dfrac{1}{n_h} + \dfrac{1}{n_g}}$

- Tukey: $\bar{Y}_{h\bullet} - \bar{Y}_{g\bullet} \quad \pm \quad \dfrac{q_{k,n-k,\alpha}}{\sqrt{2}} S_p \sqrt{\dfrac{1}{n_h} + \dfrac{1}{n_g}}$