

Datos $(X_1, Y_1), \dots, (X_n, Y_n)$

Estimadores

$$\begin{aligned}\hat{\beta}_0 &= \bar{Y} - \hat{\beta}_1 \bar{X}, \\ \hat{\beta}_1 &= \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\Delta_X}, \quad \text{donde } \Delta_X = \sum_{i=1}^n (X_i - \bar{X})^2 \\ \hat{\sigma}^2 &= S^2 = \frac{1}{n-2} \sum_{i=1}^n (Y_i - (\hat{\beta}_0 + \hat{\beta}_1 X_i))^2.\end{aligned}$$

Distribuciones muestrales: bajo los supuestos del modelo,

$$\hat{\beta}_0 \sim N\left(\beta_0, \sigma^2 \left(\frac{1}{n} + \frac{\bar{X}^2}{\Delta_X}\right)\right), \quad \hat{\beta}_1 \sim N\left(\beta_1, \frac{\sigma^2}{\Delta_X}\right)$$

y además

$$\frac{\hat{\beta}_0 - \beta_0}{\sqrt{\hat{\sigma}^2 \left(\frac{1}{n} + \frac{\bar{X}^2}{\Delta_X}\right)}} \sim t_{n-2}, \quad \frac{\hat{\beta}_1 - \beta_1}{\sqrt{\frac{\hat{\sigma}^2}{\Delta_X}}} \sim t_{n-2}$$

Predicción

$$\begin{aligned}\hat{Y}_h &\sim N\left(\beta_0 + \beta_1 X_h, \sigma^2 \left(\frac{1}{n} + \frac{(X_h - \bar{X})^2}{\Delta_X}\right)\right) \\ Y_h - \hat{Y}_h &\sim N\left(0, \sigma^2 \left(1 + \frac{1}{n} + \frac{(X_h - \bar{X})^2}{\Delta_X}\right)\right)\end{aligned}$$

y además

$$\frac{\hat{Y}_h - (\beta_0 + \beta_1 X_h)}{\sqrt{\hat{\sigma}^2 \left(\frac{1}{n} + \frac{(X_h - \bar{X})^2}{\Delta_X}\right)}} \sim t_{n-2}, \quad \frac{Y_h - \hat{Y}_h}{\sqrt{\hat{\sigma}^2 \left(1 + \frac{1}{n} + \frac{(X_h - \bar{X})^2}{\Delta_X}\right)}} \sim t_{n-2}$$

Regresión inversa - Calibración

$$\frac{\hat{X}_{nue} - X_{nue}}{\sqrt{\frac{\hat{\sigma}^2}{\hat{\beta}_1^2} \left(1 + \frac{1}{n} + \frac{(\hat{X}_{nue} - \bar{X})^2}{\Delta_X}\right)}} \stackrel{a}{\sim} t_{n-2}$$