Repaso

Proba (C)-2015

Desigualdad de Markov

$$Y \geq 0$$

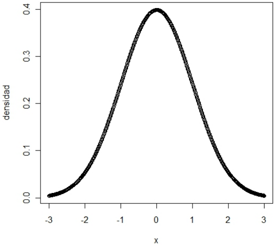
$$P(Y \geq \lambda) \leq \frac{E[Y]}{\lambda}$$

Desigualdad de Tchebishev

$$P(|X - \mu_X| \geq \varepsilon) \leq \frac{V(X)}{\varepsilon^2} \quad \mu_X = E[X]$$

LA normal

$$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

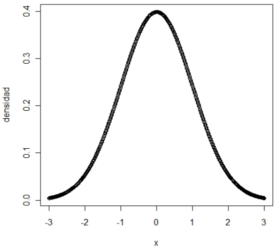


Distribucion Normal

- Z normal estandart si

$$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

- f_Z simétrica en el origen: $f_Z(z) = f_Z(-z)$
- Siendo f_Z simétrica, tenemos que $F_Z(-u) = 1 - F_Z(u)$
- $F_Z(z) = \int_{-\infty}^{z} f_Z(u)du$ no se puede calcular.
- Hay tabla con valores de $F_Z(u)$ para $u > 0$.
- $\phi(z) = F_Z(z)$ se llama función phi.
- $E[Z] = 0$, $V(Z) = 1$.


densidad
0.0
0.1
0.2
0.3
0.4
-3
-2
-1
0
1
2
3
x

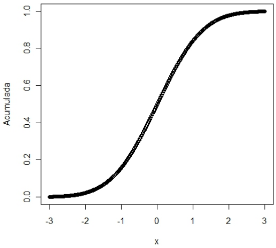

Acumulada
0.0
0.2
0.4
0.6
0.8
1.0
-3
-2
-1
0
1
2
3
x

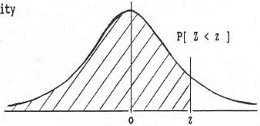
Tabla Normal

STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z i.e.

$$P[\, Z < z \,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952

Algunas cuentas

- Calcule $\phi(1.3)$
- Calcule $\phi(-1.3)$
- Encuentre el percentil 0.9
- Encuentre el percentil 0.05
- Encuentre z_0 tal que $P(|Z| \leq z_0) = 0.90$
- Encuentre z_1 tal que $P(|Z| \leq z_1) = 0.95$
- Encuentre z_2 tal que $P(|Z| \leq z_2) = 0.99$

Normal $\mathcal{N}(\mu, \sigma^2)$

- Z normal estandar, Sea $X := \sigma Z + \mu$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(x-\mu)^2}{2\sigma^2}}$$

- $E[X] = \mu$, $V(X) = \sigma^2$
- $F_X(x) = \phi((x - \mu)/\sigma)$
- $X \sim \mathcal{N}(\mu, \sigma^2)$

Normal $\mathcal{N}(\mu, \sigma^2)$

- $X \sim \mathcal{N}(\mu, \sigma^2)$, $\mu \in \mathbb{R}$, $\sigma > 0$ tiene densidad

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(x-\mu)^2}{2\sigma^2}}$$

- $E[X] = \mu,\ V(X) = \sigma^2$
- X normal con media μ y desvío σ (o varianza σ^2) : $X \sim \mathcal{N}(\mu, \sigma^2)$.
- $dnorm(x, mu, sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(x-\mu)^2}{2\sigma^2}}$
- $pnorm(x, mu, sigma) = P(X \leq x)$.

Estandarización

- $X \sim N(5, 9)$. Calcule la probabilidad de que X diste de su media en más de un desvío.
- $X \sim N(5, 9)$. Calcule la probabilidad de que X diste de su media más de 0.5.

Concentracion

- Sea $X_n \sim \mathcal{N}(5, 9/n)$. Obtenga una expresión (en términos de ϕ) para la probabilidad de que X_n diste de su media más de 0.5.
- Encuentre n de forma que la probabilidad obtenida sea menor a 0.1
- Calcule el límite de la expresión obtenida, cuando $n \to \infty$.