

Discretización de derivadas

$f'(a)$

- **2 PUNTOS.** Error $O(h)$

– Forward:

$$\frac{1}{h}(f(a+h) - f(a))$$

– Forward:

$$\frac{1}{h}(f(a) - f(a-h))$$

- **3 PUNTOS.** Error $O(h^2)$

– Forward:

$$\frac{1}{2h}(-f(a+2h) + 2f(a+h) - 3f(a))$$

– Centrada:

$$\frac{1}{2h}(f(a+h) - f(a-h))$$

– Backward:

$$\frac{1}{2h}(f(a-2h) - 2f(a-h) + 3f(a))$$

- **4 PUNTOS.** Error $O(h^3)$

– Forward:

$$\frac{1}{6h}(2f(a+3h) - 9f(a+2h) + 18f(a+h) - 11f(a))$$

– Backward:

$$\frac{1}{6h}(-2f(a-3h) + 9f(a-2h) - 18f(a-h) + 11f(a))$$

- **5 PUNTOS.** Error $O(h^4)$

– Forward:

$$\frac{1}{12h}(-3f(a+4h) + 16f(a+3h) - 36f(a+2h) + 48f(a+h) - 25f(a))$$

– Centrada:

$$\frac{1}{12h}(-f(a+2h) + 8f(a+h) - 8f(a-h) + f(a-2h))$$

– Backward:

$$\frac{1}{12h}(3f(a-4h) - 16f(a-3h) + 36f(a-2h) - 48f(a-h) + 25f(a))$$

$f''(a)$

• **3 - 4 PUNTOS.** Error $O(h^2)$

– Forward:

$$\frac{1}{h^2}(2f(a+3h) - 5f(a+2h) + 4f(a+h) - f(a))$$

– Centrada:

$$\frac{1}{h^2}(f(a+h) - 2f(a) + f(a-h))$$

– Backward:

$$\frac{1}{h^2}(2f(a-3h) - 5f(a-2h) + 4f(a-h) - f(a))$$

• **5 PUNTOS.** Error $O(h^3)$

– Forward:

$$\frac{1}{12h}(11f(a+4h) - 56f(a+3h) + 114f(a+2h) - 104f(a+h) + 35f(a))$$

– Backward:

$$\frac{1}{12h}(11f(a-4h) - 56f(a-3h) + 114f(a-2h) - 104f(a-h) + 35f(a))$$

• **5 - 6 PUNTOS.** Error $O(h^4)$

– Forward:

$$\frac{1}{12h}(-10f(a+5h) + 61f(a+4h) - 156f(a+3h) + 214f(a+2h) - 144f(a+h) + 45f(a))$$

– Centrada:

$$\frac{1}{12h}(-f(a+2h) + 16f(a+h) - 30f(a) + 16f(a-h) - f(a-2h))$$

– Backward:

$$\frac{1}{12h}(-10f(a-5h) + 61f(a-4h) - 156f(a-3h) + 214f(a-2h) - 144f(a-h) + 45f(a))$$