$$A = \begin{pmatrix} 3 & 0 & 0 & 0 & -1 & 1 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & -1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 & 3 & 0 \\ 1 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

$$\det(A-\lambda I) = \det \begin{pmatrix} 3-\lambda & 0 & 0 & 0 & -1 & 1 \\ 0 & 3-\lambda & 0 & 0 & 0 & 0 \\ 4 & 0 & -1-\lambda & 0 & -1 & 1 \\ 0 & 1 & 0 & 3-\lambda & 0 & 0 \\ 1 & 0 & 0 & 0 & 3-\lambda & 0 \\ 1 & 0 & 0 & 0 & 0 & 3-\lambda \end{pmatrix}$$

$$= (3-\lambda)^2(-1-\lambda) \det \begin{pmatrix} 3-\lambda & -1 & 1 \\ 1 & 3-\lambda & 0 \\ 1 & 0 & 3-\lambda \end{pmatrix} \quad c_2 + c_3 \to c_3$$

$$= (3-\lambda)^2(-1-\lambda) \det \begin{pmatrix} 3-\lambda & -1 & 0 \\ 1 & 3-\lambda & 3-\lambda \\ 1 & 0 & 3-\lambda \end{pmatrix}$$

$$= (3-\lambda)^3(-1-\lambda) \det \begin{pmatrix} 3-\lambda & -1 & 0 \\ 1 & 3-\lambda & 1 \\ 1 & 0 & 1 \end{pmatrix} = (3-\lambda)^3(-1-\lambda) \det \begin{pmatrix} 3-\lambda & -1 & 0 \\ 1 & 3-\lambda & 1 \\ 0 & \lambda-3 & 0 \end{pmatrix}$$

$$= (3-\lambda)^4(\lambda+1) \underbrace{\det \begin{pmatrix} 3-\lambda & -1 & 0 \\ 1 & 3-\lambda & 1 \\ 0 & 1 & 0 \end{pmatrix}}_{-\det \begin{pmatrix} 3-\lambda & 0 \\ 1 & 1 \end{pmatrix}} = -(3-\lambda)^5(\lambda+1)$$

$$\chi_A(\lambda) = (\lambda-3)^5 \underbrace{(\lambda+1)}$$

autov $\lambda_1 = -1$

$\lambda_2 = 3$ multip 5 !

$\lambda = -1$:

$$A+I = \begin{pmatrix} 4 & 0 & 0 & 0 & -1 & 1 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ \cancel{4} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{-1} & \cancel{1} \\ 0 & 1 & 0 & 4 & 0 & 0 \\ 1 & 0 & 0 & 0 & 4 & 0 \\ 1 & 0 & 0 & 0 & 0 & 4 \end{pmatrix} \to \begin{pmatrix} 4 & 0 & 0 & 0 & -1 & 1 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 \\ 0 & 0 & 0 & 0 & 17 & -1 \\ 0 & 0 & 0 & 0 & -4 & 4 \end{pmatrix}$$

$x_5 = 0 \quad x_6 = 0 \quad x_4 = 0 \quad x_2 = 0 \quad 4x_1 - x_5 + x_6 = 0$

x_3 libre $\quad S_{\lambda=-1} = \langle e_3 \rangle$ Se ve de la matriz que $e_3 \in N(A+I)$ y además tiene dim 1 ✓

$\lambda = 3$:

$$A-3I = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & -4 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ \cancel{1} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

$\mathrm{Rg}\,(A-3I) = 4 \Rightarrow \dim N(A-3I) = 2.$

$x_1 = 0 \quad x_2 = 0 \quad x_5 = x_6 \quad x_3 = 0$

$(0,0,0,x_4,x_5,x_5) = x_4 e_4 + x_5 (e_5 + e_6)$

$$\boxed{N(A-3I) = \langle e_4, e_5 + e_6 \rangle}$$

$\dim N(A-3I) = 2$
$\to$ 2 bloques para $\lambda_2 = 3$.

$\cancel{A-3I \quad (A-3I)^2}$

$\cancel{e_1 \to 4e_3+e_5+e_6 \to}$

$$(A-3I)(A-3I)=\begin{pmatrix}0&0&0&0&-1&1\\0&0&0&0&0&0\\4&0&-4&0&-1&1\\0&1&0&0&0&0\\1&0&0&0&0&0\\1&0&0&0&0&0\end{pmatrix}\begin{pmatrix}0&0&0&0&-1&1\\0&0&0&0&0&0\\4&0&-4&0&-1&1\\0&1&0&0&0&0\\1&0&0&0&0&0\\1&0&0&0&0&0\end{pmatrix}$$

$$=\begin{pmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\-16&0&16&0&0&0\\0&0&0&0&0&0\\0&0&0&0&-1&1\\0&0&0&0&-1&1\end{pmatrix}$$

$¿N(A-3I)?$ $\quad Rg\left((A-3I)^2\right)=2 \Rightarrow \dim N(A-3I)^2=4$

$$\begin{cases}-x_1+x_3=0 & x_3=x_1\\ -x_5+x_6=0 & x_5=x_6\end{cases}$$

$(x_1,x_2,x_1,x_4,x_5,x_5)$
$=x_1\cdot(e_1+e_3)$
$+\,x_2\cdot e_2+x_4\cdot e_4+x_5(e_5+e_6)$

$N(A-3I)^2=\langle e_1+e_3,\ e_2,\ e_4,\ e_5+e_6\rangle$

$$(A-3I)^3=(A-3I)^2(A-3I)=\begin{pmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\64&0&-64&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{pmatrix}$$

$Rg\left((A-3I)^3\right)=1 \Rightarrow \dim N(A-3I)^3=5$

$x_1-x_3=0 \quad x_3=x_1 \quad N\left((A-3I)^3\right)=\langle e_2,e_4,e_5,e_6,\ e_1+e_3\rangle$

$$(A-3I)^4=(A-3I)^3(A-3I)=\begin{pmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\-4\cdot 64&0&+4\cdot 64&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{pmatrix}$$

$Rg(A-3I)^4=1 \Rightarrow \dim N(A-3I)^4=5\,!$

Se estabilizó!

$$\mathcal{N}(A-3I) \subsetneq \mathcal{N}(A-3I)^2 \subsetneq \mathcal{N}(A-3I)^3 = \mathcal{N}(A-3I)^4 = \dots$$

dim: $\quad 2 \qquad\qquad 4 \qquad\qquad 5$

$$\bullet \text{———} \bullet \text{———} \bullet$$
$$(A-3I)^2 v_1 \qquad (A-3I)v_1 \qquad v_1$$

$$\bullet \text{———} \bullet$$
$$(A-3I)v_2 \qquad v_2$$

1º) Completo base de $\mathcal{N}(A-3I)^2$ a base de $\mathcal{N}(A-3I)^3$

$$\mathcal{N}(A-3I)^2 = \langle e_1+e_3,\ e_2,\ e_4,\ e_5+e_6 \rangle$$

$$\mathcal{N}(A-3I)^3 = \langle e_2,\ e_4,\ e_5,\ e_6,\ e_1+e_3 \rangle$$

$$\mathcal{N}(A-3I)^3 = \underbrace{\langle e_1+e_3,\ e_2,\ e_4,\ e_5+e_6 \rangle}_{\mathcal{N}(A-3I)^2} \oplus \langle e_5 \rangle$$
(ó e_6!)

$$\Rightarrow \boxed{v_1 = e_6}$$

$$(A-3I)e_6 = e_1+e_3$$
$$(A-3I)^2 e_6 = \underline{e_5+e_6} \in \mathcal{N}(A-3I)$$
$$(A-3I)^3 e_6 = 0$$

2º) Completo base de $\mathcal{N}(A-3I)$ a base de $\mathcal{N}(A-3I)^2$

$$\mathcal{N}(A-3I)^2 = \underbrace{\langle e_4,\ e_5+e_6 \rangle}_{\mathcal{N}(A-3I)} \oplus \underbrace{\langle \overset{e_1+e_3}{(A-3I)e_6} \rangle}_{\text{paso ant.}} \oplus \underbrace{\langle e_2 \rangle}_{v_2!}$$

$$\underline{(A-3I)e_2 = e_4}$$

Base: $B_J = \{ v_1, (A-3I)v_1, (A-3I)^2 v_1, v_2, (A-3I)v_2, v_3 \}$

$$|f|_{B_J} = \begin{pmatrix} 3 & 0 & 0 & & & \\ 1 & 3 & 0 & & & \\ 0 & 1 & 3 & & & \\ & & & 3 & 0 & \\ & & & 1 & 3 & \\ & & & & & -1 \end{pmatrix}$$