

A general limit lifting theorem for 2-dimensional monad theory

Martin Szyld

2017

Weak morphisms of T -algebras

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($\mathcal{K} \xrightarrow{T} \mathcal{K}$, $id \xrightarrow{i} T$ unit, $T^2 \xrightarrow{m} T$ multiplication)

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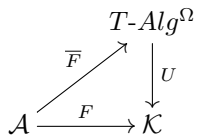
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Considering $\Omega_\ell = \text{2-cells}(\mathcal{K})$, $\Omega_p = \{\text{invertible 2-cells}\}$,
 $\Omega_s = \{\text{identities}\}$, we recover the three cases above.

Limit lifting along the forgetful functor



U creates limits \equiv we can give $\lim F$ a T -algebra structure such that it is $\lim \bar{F}$
(we *lift* the limit of F along U)

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Previous results

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Note: all these limits are *weighted* by another 2-functor $\mathcal{A} \xrightarrow{W} \mathcal{Cat}$.
Also, the projections of the limit are always strict algebra morphisms.

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We will present a theorem which unifies and generalizes these results.

A general notion of weighted limit. The conical case (Gray)

We fix \mathcal{A}, \mathcal{B} 2-categories, $\Sigma \subseteq \text{Arrows}(\mathcal{A})$, $\Omega \subseteq \text{2-cells}(\mathcal{B})$

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- σ - ω -natural transformation: $\mathcal{A} \begin{array}{c} \xrightarrow{F} \\ \theta \Downarrow \\ \xrightarrow{G} \end{array} \mathcal{B}$, θ is a lax natural

transformation
$$\begin{array}{ccc} FA & \xrightarrow{\theta_A} & GA \\ Ff \downarrow & \Downarrow \theta_f & \downarrow Gf \\ FB & \xrightarrow{\theta_B} & GB \end{array}$$
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- σ - ω -cone (for F , with vertex $E \in \mathcal{B}$): is a σ - ω -natural $\mathcal{A} \begin{array}{c} \xrightarrow{\Delta E} \\ \theta \Downarrow \\ \xrightarrow{F} \end{array} \mathcal{B}$,

i.e.
$$\begin{array}{ccc} & & FA \\ & \nearrow \theta_A & \downarrow Ff \\ E & \Downarrow \theta_f & \\ & \searrow \theta_B & FB \end{array}$$
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- σ - ω -limit: is the universal σ - ω -cone, in the sense that the following is an isomorphism

$$\mathcal{B}(E, L) \xrightarrow{\pi_*} \sigma\text{-}\omega\text{-Cones}(E, F)$$

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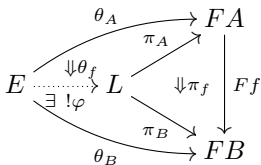
On objects: $\varphi \longleftrightarrow \theta$

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$\exists ! \varphi$

- We have the dual notions of σ - ω -opnatural, σ - ω -oplmit, where the direction of the 2-cells is reversed.

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$\uparrow \theta_f$ (dotted arrow from E to L)
 $\exists ! \varphi$ (dotted arrow from E to L)

- We have the dual notions of σ - ω -opnatural, σ - ω -oplmit, where the direction of the 2-cells is reversed.
- As with weak morphisms, the notions of lax, pseudo and strict limits are recovered with particular choices of Ω (and Σ).

Our limit lifting theorem (finding the hypotheses)

We consider $\Sigma \subseteq \text{Arrows}(\mathcal{A})$, $\Omega, \Omega' \subseteq 2\text{-cells}(\mathcal{K})$. The σ - ω -limits are always taken with respect to Σ and Ω .

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$\theta_f \in \Omega$ if $f \in \Sigma$: $T(\Omega) \subseteq \Omega$, $\Omega' \subseteq \Omega \Rightarrow TL \xrightarrow{\ell} L$.

The limit L is Ω' -compatible $\Rightarrow (TL, \ell)$ is the desired lifted limit.

Our limit lifting theorem (properly stated)

Theorem: Let $\Sigma \subseteq \text{Arrows}(\mathcal{A})$, $\Omega, \Omega' \subseteq 2\text{-cells}(\mathcal{K})$. Assume $T(\Omega) \subseteq \Omega$ and $\Omega' \subseteq \Omega$. Then $T\text{-Alg}^{\Omega'} \xrightarrow{U} \mathcal{K}$ creates Ω' -compatible σ - ω -oplimits.

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The case $\Omega, \Omega' \in \{\Omega_\ell, \Omega_p, \Omega_s\}$

$T(\Omega) \subseteq \Omega$ ✓, Ω' -compatible ✓

- ① (with $\Omega = \Omega' = \Omega_\ell$) $T\text{-Alg}_\ell \xrightarrow{U} \mathcal{K}$ creates oplax limits.
- ② (with $\Omega = \Omega' = \Omega_p$) $T\text{-Alg}_p \xrightarrow{U} \mathcal{K}$ creates σ -limits (thus in particular lax and pseudolimits).
- ③ (with $\Omega = \Omega' = \Omega_s$) $T\text{-Alg}_s \xrightarrow{U} \mathcal{K}$ creates all (strict) limits.

Present and future work

- The 2-category $Hom_{\sigma,\omega}(F, G)$ as a 2-category of weak morphisms.
- More examples like that one, in which Ω is not one of $\Omega_{\ell,p,s}$ (may arise from weak equivalences?)
- Bilimit lifting (projections probably won't be strict).
- Other results from 2-dimensional monad theory (flexibility, biadjunctions).