A general limit lifting theorem for 2-dimensional monad theory

Martin Szyld

2017

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Considering $\Omega_\ell =$ 2-cells($\mathcal{K}$), $\Omega_p = \{$invertible 2-cells$\}$, $\Omega_s = \{$identities$\}$, we recover the three cases above.

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Note: all these limits are *weighted* by another 2-functor $\mathcal{A} \xrightarrow{W} \mathcal{C}at$.
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We will present a theorem which unifies and generalizes these results.

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- σ-ω-natural transformation: $\mathcal{A} \underset{G}{\overset{F}{\underset{\theta\Downarrow}{\rightrightarrows}}} \mathcal{B}$, θ is a lax natural transformation
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such that θ_f is in Ω when f is in Σ.

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- σ-ω-cone (for F, with vertex $E \in \mathcal{B}$): is a σ-ω-natural $\mathcal{A} \underset{F}{\overset{\triangle E}{\underset{\theta\Downarrow}{\rightrightarrows}}} \mathcal{B}$,

i.e.
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• We have the dual notions of σ-ω-opnatural, σ-ω-oplimit, where the direction of the 2-cells is reversed.

• As with weak morphisms, the notions of lax, pseudo and strict limits are recovered with particular choices of Ω (and Σ).

Our limit lifting theorem (finding the hypotheses)

We consider $\Sigma \subseteq \text{Arrows}(\mathcal{A})$, $\Omega, \Omega' \subseteq \text{2-cells}(\mathcal{K})$. The σ-ω-limits are always taken with respect to Σ and Ω.

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$\theta_f \in \Omega$ if $f \in \Sigma$: $\quad T(\Omega) \subseteq \Omega$, $\Omega' \subseteq \Omega \Rightarrow TL \xrightarrow{\ell} L$.

The limit L is Ω'-compatible $\Rightarrow (TL, \ell)$ is the desired lifted limit.

Our limit lifting theorem (properly stated)

Theorem: Let $\Sigma \subseteq \text{Arrows}(\mathcal{A})$, $\Omega, \Omega' \subseteq \text{2-cells}(\mathcal{K})$. Assume $T(\Omega) \subseteq \Omega$ and $\Omega' \subseteq \Omega$. Then $T\text{-}Alg^{\Omega'} \xrightarrow{U} \mathcal{K}$ creates Ω'-compatible σ-ω-oplimits.

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The case $\Omega, \Omega' \in \{\Omega_\ell, \Omega_p, \Omega_s\}$

$T(\Omega) \subseteq \Omega$ ✓, Ω'-compatible ✓

1. (with $\Omega = \Omega' = \Omega_\ell$) $T\text{-}Alg_\ell \xrightarrow{U} \mathcal{K}$ creates oplax limits.
2. (with $\Omega = \Omega' = \Omega_p$) $T\text{-}Alg_p \xrightarrow{U} \mathcal{K}$ creates σ-limits (thus in particular lax and pseudolimits).
3. (with $\Omega = \Omega' = \Omega_s$) $T\text{-}Alg_s \xrightarrow{U} \mathcal{K}$ creates all (strict) limits.

Present and future work

- The 2-category $Hom_{\sigma,\omega}(F, G)$ as a 2-category of weak morphisms.
- More examples like that one, in which Ω is not one of $\Omega_{\ell,p,s}$ (may arise from weak equivalences?)
- Bilimit lifting (projections probably won't be strict).
- Other results from 2-dimensional monad theory (flexibility, biadjunctions).