Persistence and periodic solutions in systems of delay differential equations

Melanie Bondorevsky
Departamento de Matemática - FCEyN
Universidad de Buenos Aires & IMAS-CONICET

Abstract

We study semi-dynamical systems associated to delay differential equations. With population models in mind, we consider the delayed differential system

$$x'(t) = f(t, x(t), x(t - \tau)) \tag{1}$$

where $f : \mathbb{R} \times [0, +\infty)^{2N} \to \mathbb{R}^N$ is continuous and $\tau \in \mathbb{R}^+$ is the delay. An initial condition for (1) can be expressed in the following way

$$x_0 = \varphi, \tag{2}$$

where $\varphi : [-\tau, 0] \to [0, +\infty)^N$ is a continuous function and $x_t \in C([-\tau, 0], \mathbb{R}^N)$ is defined by $x_t(s) = x(t + s)$. Thus, the flow

$$\Phi : [0, +\infty) \times C([-\tau, 0], \mathbb{R}^N) \to C([-\tau, 0], \mathbb{R}^N), \tag{3}$$

given by $\Phi(t, \varphi) = x_t$, induces a semi-dynamical system. We give sufficient conditions to guarantee uniform persistence, employing guiding functions techniques.

In order to find periodic orbits of (3) we employ topological degree methods. Since the space of initial conditions is infinite dimensional, the Brouwer degree cannot be applied: we instead use Leray-Schauder degree techniques. More precisely, inspired by Krasnoselskii's work, we shall consider the positive cone X of C_T, the Banach space of continuous T-periodic functions, for some $T > 0$, and define an appropriate fixed point operator $K : X \to C_T$.

The results are based on the work *Persistence and periodic solutions in systems of delay differential equations* (to be submitted), of P. Amster and M. Bondorevsky.