The obstacle problem for a degenerate fully nonlinear operator

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Joint work with João Vitor Da Silva.

We study the obstacle problem for fully nonlinear elliptic operators with an anisotropic degeneracy on the gradient:

$$\begin{cases} \min\left\{f - |Du|^{\gamma} F(D^2u), u - \phi\right\} = 0 & \text{in } \Omega \\ u = g & \text{on } \partial\Omega. \end{cases}$$

We obtain existence of solutions and prove sharp regularity estimates along the free boundary points, namely $\partial\{u > \phi\} \cap \Omega$. In particular, for the homogeneous case ($f \equiv 0$) we get that solutions are $C^{1,1}$ at free boundary points, in the sense that they detach from the obstacle in a quadratic fashion, thus beating the optimal regularity allowed for such degenerate operators. We also present further features of the solutions and a partial result regarding the free boundary.

These are the first results for obstacle problems driven by degenerate type operators in non-divergence form. Finally, they are a novelty even for the simpler scenario given by an operator of the form $\mathcal{G}[u] = |Du|^{\gamma} \Delta u$.