

**Control and Observability of the Heat Equation:
An Introduction to the Control of
Partial Differential Equations**

A control system is a dynamical system on which we can act by means of a *control*. In the context of partial differential equations (PDE), a controllability problem consists in driving the solution of the PDE to a final target, at a given time, by choosing an appropriate control which acts on part of the domain or on its boundary.

In this double-session course we will introduce the control theory of PDE by analyzing controllability properties of the heat equation. In particular, we will see that, under a suitable functional setting, controllability is equivalent to an observability problem for a heat equation without control. Then, we will present one of the most powerful tools to prove observability: Carleman estimates.

**Control y Observabilidad de la Ecuación del Calor:
Una Introducción al Control de
Ecuaciones en Derivadas Parciales**

Un sistema de control es un sistema dinámico sobre el cual podemos actuar por medio de un *control*. En el contexto de ecuaciones en derivadas parciales (EDP), un problema de control consiste en llevar la solución de una EDP a un estado final, en un tiempo final dado, escogiendo un control apropiado que actúa en parte del dominio o de su frontera.

En este curso de dos sesiones introduciremos la teoría de control de EDP analizando propiedades de controlabilidad de la ecuación del calor. En particular, veremos que, en un marco funcional apropiado, la controlabilidad es equivalente a un problema de observabilidad para una ecuación del calor sin control. Luego, presentaremos una de las herramientas más poderosas para probar observabilidad: estimaciones de Carleman.

References

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