Multiple solutions for periodic perturbations of a delayed autonomous system near an equilibrium

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In this work, we study small non-autonomous perturbations around an equilibrium of the following nonlinear delayed system

$$u'(t) = g(u(t), u(t-\tau)) + p(t),$$

where $\tau > 0$, $g : \overline{\Omega} \times \overline{\Omega} \to \mathbb{R}^N$ is continuously differentiable, p is continuous and $T-$periodic and $\Omega \subset \mathbb{R}^N$.

Under appropriate assumptions, it is shown that the number of T-periodic solutions lying inside the bounded domain Ω is, generically, at least $|\chi \pm 1| + 1$, where χ denotes the Euler characteristic of Ω.

Moreover, some connections between the associated fixed point operator and the Poincaré operator are explored.

References

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