ERRATA: H^2 REGULARITY FOR THE $p(x)-$LAPLACIAN IN TWO-DIMENSIONAL CONVEX DOMAINS

LEANDRO M. DEL PEZZO AND SANDRA MARTÍNEZ

The statements of Theorem 2.4, Lemma 4.1 and Remark 5.1 (3) in [1] are incorrect, the correct statements are the following. Also for the Lemma 4.1, we need to change the proof.

Theorem 1 (The correct statement of Theorem 2.4 in [1])**.** *Let Ω be a Lipschitz domain. Let $p:\Omega\to[1,p_2]$ and p be a log-Hölder continuous with $p_2<N$. Then the embedding $W^{1,p(\cdot)}(\Omega)\hookrightarrow L^{p^*(\cdot)}(\Omega)$ is continuous.*

Proof. See [2, Corollary 8.3.2]. ☐

Lemma 2 (The correct statement of Lemma 4.1 in [1])**.** *Let Ω be a Lipschitz domain, $p\colon\Omega\to[1,+\infty)$ be a $\log-$Hölder function, $f\in L^{q(\cdot)}(\Omega)$ with $q(x)\geq q_1>(p_1^*)'$, $g\in W^{1,p(\cdot)}(\Omega)$, $\varepsilon\geq0$ and u_ε be the weak solution of*

$$\begin{cases} -\operatorname{div}\left((\varepsilon+|\nabla u|^2)^{\frac{p(x)-2}{2}}\nabla u\right)=f & \text{in } \Omega,\\ u=g & \text{on } \partial\Omega, \end{cases} \tag{1}$$

then

$$\|\nabla u_\varepsilon\|_{L^{p(\cdot)}(\Omega)}\leq C$$

where C is a constant depending only on $\|g\|_{W^{1,p(\cdot)}(\Omega)}$, $\|f\|_{L^{q(\cdot)}(\Omega)}$, $|\Omega|$, the Lipschitz constant of Ω and p_1 but not on ε.

Proof. Let

$$J(v):=\int_\Omega\frac{1}{p(x)}(|\nabla v|^2+\varepsilon)^{p(x)/2}\,dx,$$

By the convexity of J and using (1) we have that,

$$\begin{aligned} J(u_\varepsilon)&\leq J(g)-\int_\Omega(|\nabla u_\varepsilon|^2+\varepsilon)^{(p(x)-2)/2}\nabla u_\varepsilon(\nabla g-\nabla u_\varepsilon)\,dx\\ &=J(g)-\left(\int_\Omega f(g-u_\varepsilon)\,dx\right). \end{aligned}$$

Key words and phrases. Variable exponent spaces. Elliptic Equations. H^2 regularity.
2010 *Mathematics Subject Classification.* 35B65, 35J60, 35J70.
Supported by UBA X117, UBA 20020090300113, CONICET PIP 2009 845/10 and PIP 11220090100625.

Using Hölder inequality, that $q'(x) \leq q_1' < p_1^*$, $W^{1,p_1}(\Omega) \hookrightarrow L^{q_1'}(\Omega)$ continuously and Poincare inequality, we have that

$$\begin{aligned} J(u_\varepsilon) &\leq \Big(J(g) + 2\|f\|_{L^{q(\cdot)}(\Omega)}\|u_\varepsilon - g\|_{L^{q'(\cdot)}(\Omega)}\Big) \\ &\leq \Big(J(g) + C\|f\|_{L^{q(\cdot)}(\Omega)}\|u_\varepsilon - g\|_{L^{q_1'}(\Omega)}\Big) \\ &\leq \Big(J(g) + C\|f\|_{L^{q(\cdot)}(\Omega)}\|\nabla u_\varepsilon - \nabla g\|_{L^{p_1}(\Omega)}\Big) \\ &\leq C\Big(1 + \|f\|_{L^{q(\cdot)}(\Omega)}\|\nabla u_\varepsilon - \nabla g\|_{L^{p(\cdot)}(\Omega)}\Big) \end{aligned}$$

where the constant $C = C(\|g\|_{W^{1,p(\cdot)}(\Omega)}, \|f\|_{L^{q(\cdot)}(\Omega)}, |\Omega|, \text{Lip}(\Omega), p_1)$.

Thus we have that there exist a constant independent of ε such that,

$$\int_\Omega |\nabla u_\varepsilon|^{p(x)}\,dx \leq C(1 + \|\nabla u_\varepsilon\|_{L^{p(\cdot)}(\Omega)}),$$

and using the properties of the $L^{p(\cdot)}(\Omega)-$norms this means that

$$\|\nabla u_\varepsilon\|^m_{L^{p(\cdot)}(\Omega)} \leq C(1 + \|\nabla u_\varepsilon\|_{L^{p(\cdot)}(\Omega)}),$$

for some $m > 1$. Therefore $\|\nabla u_\varepsilon\|_{L^{p(\cdot)}(\Omega)}$ is bounded independent of ε. □

Remark 3 (The correct statement of Remark 5.1 (3) in [1]). Let Ω be a convex set. Then, there exists a sequence $\{\Omega_m\}_{m\in\mathbb{N}}$ of convex subset of Ω with C^2 boundary such that $\Omega_m \subset \Omega_{m+1}$ for any $m \in \mathbb{N}$ and $|\Omega \setminus \Omega_m| \to 0$.

By Remark 5.1 (2) in [1] and by the classical Sobolev embedding theorem, we have that for $r > 1$ there exists a constant C independent of m such that

$$\|v\|_{L^q(\Omega_m)} \leq C\|v\|_{W^{1,r}(\Omega_m)} \quad \forall v \in W^{1,r}(\Omega_m),$$

for any $q < r^*$ and $m \in \mathbb{N}$.

Remark 4. Observe that the assumption $q(x) \geq q_1 > 2$ in [1, Theorem 1.1 and Theorem 1.2], implies that $q(x) \geq q_1 > (p_1^*)'$, since $(p_1^*)' < 2$ when $N = 2$. Therefore, Lemma 2 can be applied in both theorems. Also observe that [1, Theorem 2.4] was only use in [1, Lemma 4.1 and Remark 5.1 (3)]. With these changes, now the statements are correct.

References

1. Leandro M. Del Pezzo and Sandra Martínez, *H^2 regularity for the $p(x)$-Laplacian in two-dimensional convex domains*. J. Math. Anal. Appl. 410 (2014), no. 2, 939–952. 35J62 (35B65 35J25)
2. L. Diening, P Harjulehto, P. Hästö, and M. Ruzicka, *Lebesgue and sobolev spaces with variable exponents*, Lecture Notes in Mathematics, vol. 2017, Springer-Verlag, New York, 2011.

Leandro M. Del Pezzo
CONICET and Departamento de Matemática, FCEyN, UBA,
Pabellón I, Ciudad Universitaria (1428), Buenos Aires, Argentina.
E-mail address: ldpezzo@dm.uba.ar
Web page: http://cms.dm.uba.ar/Members/ldpezzo

Sandra Martínez
IMAS-CONICET and Departamento de Matemática, FCEyN, UBA,
Pabellón I, Ciudad Universitaria (1428),Buenos Aires, Argentina.
E-mail address: smartin@dm.uba.ar