MATH 4430:
ADDITIONAL PRACTICE PROBLEMS

(1) Solve by the method of variation of parameters.

(a)
$$\dot{x} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix} x + \begin{bmatrix} e^t \\ e^{3t} \end{bmatrix}$$
Answer: $x = C_1(e^{-t}, e^{-t}, -e^{-t}) + C_2((1/2)e^{2t}, e^{2t}, e^{2t}) + C_3(e^{-2t}, -e^{-2t}, 0) + ((1/6)e^t + (3/20)e^{3t} - 2, (-1/6)e^t + (7/20)e^{3t} - 2, (-1/2)e^t + (1/4)e^{3t})$.

(b)
$$\dot{x} = \begin{bmatrix} -5 & -1 \\ 1 & -3 \end{bmatrix} x + \begin{bmatrix} e^t \\ e^{2t} \end{bmatrix},$$
$$x(0) = (119/900, 211/900)$$
Answer: $x = ((4/25)e^t - (1/36)e^{2t}, (1/25)e^t + (7/36)e^{2t})$

(c)
$$\dot{x} = \begin{bmatrix} 3 & -1/2 \\ 0 & 2 \end{bmatrix} x + \begin{bmatrix} 3/2 - t/2 - 3t^2 \\ -2t - 1 \end{bmatrix},$$
$$x(0) = (2, 3)$$
Answer: $x = (e^{2t} + e^{3t} + t^2 + t, 2e^{2t} + t + 1)$

(2) Solve by the method of Laplace transform.

(a)
$$\dot{x} = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} x + \begin{bmatrix} 3t \\ 4 \end{bmatrix},$$
$$x(0) = (2, 3)$$
Answer: $x = (-(5/4) + (13/4)\cos(2t) - 3\sin(2t), (3/2)t + 3\cos(2t) + (13/4)\sin(2t))$

(b)
$$\dot{x} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} e^t \\ e^t \end{bmatrix},$$
$$x(0) = (1, 1)$$
Answer: $x = (e^t, e^t)$.

(3) Show that the origin is an equilibrium point of each of the following systems, draw the phase portrait near the origin, and determine, if possible, whether it is stable or unstable.

(a)
$$\begin{cases} \dot{x} = x + 2y - \sin(y^2) \\ \dot{y} = -x - 3y + x(e^{x^2/2} - 1) \end{cases}$$
Answer:unstable.

(b)

$$\begin{cases} \dot{x} = -x + 3y + x^2 \sin(y) \\ \dot{y} = -x - 4y + 1 - \cos(y^2) \end{cases}$$

Answer: stable.

(c)

$$\begin{cases} \dot{x} = -2x + 8\sin^2(y) \\ \dot{y} = x - 3y + 4x^3 \end{cases}$$

Answer: stable.

(d)

$$\begin{cases} \dot{x} = 3x - 22\sin(y) + x^2 - y^3 \\ \dot{y} = \sin(x) - 5y + e^{x^2} - 1 \end{cases}$$

Answer:stable.