Math 4430: Practice problems

Second order equations

1. Find the general solution of each of the following equations.

   (a) $y" - 6y' + 9y = 0$. Answer: $y = c_1e^{3t} + c_2te^{3t}$.

   (b) $y" - 2y' + 10y = 0$. Answer: $y = c_1e^t\cos(3t) + c_2e^t\sin(3t)$.

   (c) $6y" - y' - y = 0$. Answer: $y = c_1e^{t/2} + c_2e^{-t/3}$.

2. Solve the following initial value problems.

   (a) $9y" - 12y' + 4 = 0$, $y(0) = 2$, $y'(0) = -1/3$.
   Answer: $y = t/3 - (1/2)e^{4t/3} + 5/2$.

   (b) $9y" + 6y' + 82y = 0$,$y(0) = -1$, $y'(0) = 2$.
   Answer: $y = e^{-t/3}((5/9)\sin(3t) - \cos(3t))$.

   (c) $y" + 8y' - 9y = 0$, $y(0) = 1$, $y'(0) = 0$.
   Answer: $y = (1/10)(e^{-9t} + 9e^t)$.

3. For each of the equations below, verify that the given y_1 is a solution, and find the general solution.

   (a) $(t-1)y" - ty' + y = 0$, $t > 1$, $y_1 = e^t$. Answer: $y = c_1t + c_2e^t$.

   (b) $t^2y" + ty' - y = 0$, $t > 0$, $y_1 = 1/t$. Answer: $y = c_1/t + c_2t$.

   (c) $t^2y" - t(t+2)y' + (t+2)y = 0$, $t > 0$, $y_1 = t$. Answer: $y = c_1t + c_2te^t$.

4. Find a particular solution to each of the following equations

   (a) $y" + y = t(1 + \sin(t))$. Answer: $y = (t/4)(4 + \sin(t) - t\cos(t))$.

   (b) $y" - 2y' - 3y = -3te^{-t}$. Answer: $y = (3/16)(t + 2t^2)e^{-t}$.

   (c) $2y" + 3y' + y = t^2 + 3\sin(t)$.
   Answer: $y = 14 - 6t + t^2 - (1/10)(9\cos(t) + 3\sin(t))$.