Math 4430 – Practice Problems

First Order Equations.

1. Find the general solutions of the following equations:

    (1) $y + 2t^2 + y't \ln t = 0$. Answer: $y \ln t + t^2 = C$.

    (2) $y' = \frac{y+t^3}{t}$. Answer: $y = t(t^2/2 + C)$.

    (3) $t^2 y^2 + e^{1/t} + (2yt^3 + t^2 y^3)y' = 0$. Answer: $y^2 t + y^4/4 - e^{1/t} = C$.

    (4) $ty' + y = y^2$. Answer: $y = \frac{1}{1-Ct}$ or $y = 0$.

    (5) $4ty + (t^2 - 1)y' = 0$. Answer: $y = \frac{C}{(t^2-1)^2}$.

    (6) $y' - y \tan t = \frac{1}{\cos t}$. Answer: $y = \frac{t+C}{\cos t}$.

    (7) $ty' = y + 2te^{-y/t}$. Answer: $y = t \ln(\ln(Ct^2))$.

    (8) $ty + (t+1)y' = 0$. Answer: $y = Ce^{-t}(1+t)$.

    (9) $y' - y = 2t - 3$. Answer: $y = Ce^t - 2t + 1$.

    (10) $e^t y^3 t^2 + 1 + t^3 y^2 e^t y' = 0$. Answer: $t^3 y^3/3 - e^{-t} = C$.

    (11) $ty' + (1+t)y = 3t^2 e^{-t}$. Answer: $y = \frac{t^3+C}{t}e^{-t}$

    (12) $3t^2(1 + \ln y) = \left(2y - \frac{t^3}{y}\right)y'$. Answer: $t^3(1 + \ln y) - y^2 = C$

    (13) $1 + ty + t(t+y)y' = 0$. Answer: $\ln|t| + ty + y^2/2 = C$

2. Solve the following initial value problems:

    (1) $(3t^2 \ln y + e^{-t}) + \left(\frac{t^3}{y} + y\right)y' = 0$, $y(0) = 2$.

    Answer: $t^3 \ln y - e^{-t} + y^2/2 = 1$.

    (2) $y' = e^{t+y}$, $y(0) = 0$. Answer: $y = -\ln(2 - e^t)$

(3) $y^2 + ty + t^2 - tyy' = 0$, $y(2) = 2$. Answer: $e^{y/t} = \frac{e}{4}(y + t)$.

(4) $ty + e^t = ty'$, $y(1) = 1$. Answer: $y = e^t(\ln t + 1/e)$.

(5) $2t + 3y - ty' = 0$, $y(1) = 0$. Answer: $y = t^3 - t$

(6) $(t^2 + 1)y' + 2ty = \sin t$, $y(0) = 0$. Answer: $y = \frac{1 - \cos t}{t^2 + 1}$.

(7) $t - ty^2 + yy' = 0$, $y(0) = \sqrt{2}$. Answer: $y = \sqrt{1 + e^{t^2}}$.

(8) $(2ty + t^2) + (t^2 + y^2)y' = 0$, $y(0) = 3$.

Answer: $y^3/3 + yt^2 + t^3/3 = 9$.