Solutions and Answers

1. $f(t) = \frac{1}{4}e^{2t}(7\,\mathrm{sen}\,4t + 8\cos 4t)$.
2. $g(t) = 1 - e^{-t}$.
3. $h(t) = t\,\mathrm{sen}\,t$.
4. $p(t) = \frac{1}{4}e^{-2t}(-2t + e^{2t} - 1)$.
5. $q(t) = \frac{1}{(n-1)!}e^{at}t^{n-1}$.
6. $r(t) = \frac{3}{2}(\mathrm{sen}\,t + t\cos t)$.
7. $r(t) = \frac{t^4}{45\,360}(3\,780 + 54t^3 + t^5)$.
8. $q(t) = \cos(\sqrt{2}t) + \frac{1}{50}(5\sqrt{5}\,t\,\mathrm{sen}(\sqrt{5}t) - 4\sqrt{5}\,\mathrm{sen}(\sqrt{5}t) + 20t\cos(\sqrt{5}t))$.
9. $p(t) = e^{2t}\,\mathrm{sen}\,t$.
10. $h(t) = -\frac{1}{2}e^{-9t}(e^{8t} - 3)$.
11. $g(t) = \frac{12 - 7\sqrt{3}}{24}e^{(7-4\sqrt{3})t} + \frac{12 + 7\sqrt{3}}{24}e^{(7+4\sqrt{3})t}$.
12. $f(t) = \frac{1}{16}H(t-4)(1 - \cos(4t - 16))$.
13. $f(t) = \frac{1}{2}e^{5(t-1)}(t-1)^2H(t-1)$.
14. $g(t) = \frac{1}{4}(t-10)H(t-10)\,\mathrm{sen}(2t - 20)$.
15. $h(t) = -2e^t + \frac{3}{2}e^{2t} + \frac{3}{2}$.
16. $p(t) = \frac{e^{-t}}{16}(36t + 7e^{4t} - 7)$.
17. $q(t) = e^{-4t}(-4t + 5e^t - 5)$.

18. $r(t) = \dfrac{1}{25}e^{-3t}(65te^{5t} + 27e^{5t} - 2)$.

19. $r(t) = \dfrac{e^{-bt}}{2(a^2+b^2)}(-2e^{bt}\cos(at) + e^{2bt} + 1)$.

20. $q(t) = \dfrac{e^{-3t}}{6} - \dfrac{e^{-2t}}{5} + \dfrac{e^{3t}}{30}$.

21. $p(t) = \dfrac{1}{25}(5t^2 + 2\cos(\sqrt{5}\,t) - 2)$.

22. $\mathscr{L}[y' - 2y] = \mathscr{L}\left[e^{5t}\right] \Rightarrow s\mathscr{L}[y] - 3 - 2\mathscr{L}[y] = \dfrac{1}{s-5} \Rightarrow$

$$\mathscr{L}[y] = \frac{3s-14}{(s-5)(s-2)} \Rightarrow y = \frac{1}{3}e^{2t}(e^{3t}+8).$$

23. $\mathscr{L}[y'' - y] = \mathscr{L}[1] \Rightarrow s^2\mathscr{L}[y] - 1 - \mathscr{L}[y] = \dfrac{1}{s} \Rightarrow \mathscr{L}[y] = \dfrac{1}{s(s-1)} \Rightarrow y = e^t - 1.$

24. $\mathscr{L}[y'' - y' - 2y] = \mathscr{L}\left[4t^2\right] \Rightarrow s^2\mathscr{L}[y] - s - 4 - s\mathscr{L}[y] + 1 - 2\mathscr{L}[y] = \dfrac{8}{s^3}$

$$\Rightarrow \mathscr{L}[y] = \frac{8 + s^4 + 3s^3}{s^3(s^2 - s - 2)} \Rightarrow y = -2t^2 + 2t + 2e^{-t} + 2e^{2t} - 3.$$

25. $\mathscr{L}[y'' + y] = \mathscr{L}\left[e^{-2t}\operatorname{sen} t\right] \Rightarrow s^2\mathscr{L}[y] + \mathscr{L}[y] = \dfrac{1}{(s+2)^2+1}$

$$\Rightarrow \mathscr{L}[y] = \frac{1}{(s^2+1)((s+2)^2+1)} \Rightarrow y = \frac{1}{8}e^{-2t}\operatorname{sen} t + \frac{1}{8}e^{-2t}\cos t + \frac{1}{8}\operatorname{sen} t - \frac{1}{8}\cos t.$$

26. $\mathscr{L}[y''' + 4y'' + 5y' + 2y] = \mathscr{L}[10\cos t]$

$$\Rightarrow s^3\mathscr{L}[y] - 3 + 4s^2\mathscr{L}[y] + 5s\mathscr{L}[y] + 2\mathscr{L}[y] = \frac{10s}{s^2+1}$$

$$\Rightarrow \mathscr{L}[y] = \frac{3s^2 + 10s + 3}{(s+2)(s+1)^2(s^2+1)} \Rightarrow y = -2e^{-t}t - e^{-2t} + 2e^{-t} + 2\operatorname{sen} t - \cos t.$$

27. $\mathscr{L}[y'' + 4y' + 8y] = \mathscr{L}[\operatorname{sen} t] \Rightarrow s^2\mathscr{L}[y] - s + 4s\mathscr{L}[y] - 4 + 8\mathscr{L}[y] = \dfrac{1}{s^2+1}$

$$\Rightarrow \mathscr{L}[y] = \frac{s^3 + 4s^2 + s + 5}{(s^2+1)(s^2+4s+8)} \Rightarrow y = -2t^2 + 2t + 2e^{-t} + 2e^{2t} - 3.$$

28. $\mathscr{L}[y' - 2y] = \mathscr{L}[1 - t] \Rightarrow s\mathscr{L}[y] - 1 - 2\mathscr{L}[y] = \dfrac{s-1}{s^2} \Rightarrow \mathscr{L}[y] = \dfrac{s^2+s-1}{s^2(s-2)}$

$$\Rightarrow y = -\frac{1}{4} + \frac{1}{2}t + \frac{5}{4}e^{2t}.$$

29. $\mathscr{L}[y''-4y'+4y]=\mathscr{L}[1]\Rightarrow s^2\mathscr{L}[y]-s-4-4s\mathscr{L}[y]+4+4\mathscr{L}[y]=\dfrac{1}{s}$

$$\Rightarrow \mathscr{L}[y]=\frac{1+s^2}{s(s-2)^2}\Rightarrow y=\tfrac{1}{4}+\tfrac{3}{4}e^{2t}+\tfrac{5}{2}te^{2t}.$$

30. $\mathscr{L}[y''+9y]=\mathscr{L}[t]\Rightarrow s^2\mathscr{L}[y]+9\mathscr{L}[y]=\dfrac{1}{s^2}$

$$\Rightarrow \mathscr{L}[y]=\frac{1}{s^2(s^2+9)}\Rightarrow y=\tfrac{1}{9}t-\tfrac{1}{27}\operatorname{sen}3t.$$

31. $\mathscr{L}[y''-10y'+26y]=\mathscr{L}[4]\Rightarrow s^2\mathscr{L}[y]-3s-15-10s\mathscr{L}[y]+30+26\mathscr{L}[y]=\dfrac{4}{s}$

$$\Rightarrow \mathscr{L}[y]=\frac{3s^2-15s+4}{s(s^2-10s+26)}\Rightarrow y=\tfrac{2}{13}+\tfrac{10}{13}e^{5t}\operatorname{sen}t+\tfrac{37}{13}e^{5t}\cos t.$$

32. $\mathscr{L}[y''-6y'+8y]=\mathscr{L}[e^t]\Rightarrow s^2\mathscr{L}[y]-3s-9-6s\mathscr{L}[y]+18+8\mathscr{L}[y]=\dfrac{1}{s-1}$

$$\Rightarrow \mathscr{L}[y]=\frac{3s^2-12s+10}{(s-1)(s^2-6s+8)}\Rightarrow y=\tfrac{1}{3}e^t+e^{2t}+\tfrac{5}{3}e^{4t}.$$

33. $\mathscr{L}[y''+4y]=\mathscr{L}[e^{-t}\operatorname{sen}t]\Rightarrow s^2\mathscr{L}[y]-s-4+4\mathscr{L}[y]=\dfrac{1}{(s+1)^2+1}$

$$\Rightarrow \mathscr{L}[y]=\frac{s^3+6s^2+10s+9}{(s^2+4)((s+1)^2+1)}\Rightarrow y=\tfrac{1}{5}e^{-t}\operatorname{sen}t+\tfrac{1}{10}e^{-t}\cos t+\tfrac{39}{20}\operatorname{sen}2t+\tfrac{9}{10}\cos 2t.$$

34. $\mathscr{L}[y''+2y'-3y]=\mathscr{L}[e^{-3t}]\Rightarrow s^2\mathscr{L}[y]+2s\mathscr{L}[y]-3\mathscr{L}[y]=\dfrac{1}{s+3}$

$$\Rightarrow \mathscr{L}[y]=\frac{1}{(s+3)(s^2+2s-3)}\Rightarrow y=\tfrac{1}{16}e^t-\tfrac{1}{16}e^{-3t}-\tfrac{1}{4}te^{-3t}.$$

35. $f(t)=2H(t-4)\Rightarrow \mathscr{L}[f(t)]=\mathscr{L}[2H(t-4)]=\dfrac{2}{s}e^{-4s}.$

Hence
~~Luego~~, $s^3\mathscr{L}[y]-8\mathscr{L}[y]=\dfrac{2}{s}e^{-4s}\Rightarrow \mathscr{L}[y]=\dfrac{2e^{-4s}}{s(s^3-8)}$

$$\Rightarrow \frac{1}{12}H(t-4)\left(e^{2(t-4)}+2e^{4-t}\cos(\sqrt{3}(t-4))-3\right).$$

36. $f(t)=t+2H(t-3)\Rightarrow \mathscr{L}[f(t)]=\mathscr{L}[t+2H(t-3)]=\dfrac{1}{s^2}+\dfrac{2}{s}e^{-3s}.$

Hence
~~Luego~~, $s^2\mathscr{L}[y]+2s-1-4s\mathscr{L}[y]-8+4\mathscr{L}[y]=\dfrac{1}{s^2}+\dfrac{2}{s}e^{-3s}$

$$\Rightarrow \mathscr{L}[y]=\frac{2s^3+9s^2+1+2se^{-3s}}{s^2(s^2-4s+4)}$$

$$\Rightarrow y=\frac{2(e^{2t}(2t-7)+e^6)H(t-3)+e^6(t+e^{2t}(53t+7)+1)}{4e^6}.$$